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Theory of Photoelasticity in Amorphous Solids

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The photoelastic effect in amorphous solids is caused by two effects. The elastic deformations alter the Lorentz-Lorenz interaction between the dipoles, and in addition produce optical anisotropy of the atoms. For pressure the first effect creates positive birefringence, while the second effect gives negative birefringence. In most cases the second effect is larger. With increasing index of refraction the first effect increases faster than the second effect, and hence the photoelastic constants decrease and reversal of sign occurs. Measurements on glasses verify the theory. A uniform volume expansion of 1 percent increases the refraction of the oxygen ions by 0.4 percent and a strain of 1 percent creates an optical anisotropy of the 0 ion of about 0.7 percent. For the positive ions these effects are somewhat smaller.

CLASSICAL THEORY

AN optically isotropic solid becomes birefringent under the influence of pressure. No adequate theory of this effect has been given. The theory of Ambronn[1] and Wiener[2] is based on assumptions which are not valid for most solids. An article by Voigt[3] does not explain the observations and the papers of Herzfeld[4] and Banerjee[5] contain mathematical errors.[6]

The photoelastic properties of an amorphous solid are characterized by the two stress-optical constants[7] B_1 and B_2, defined by (1)

$$n_z-n=B_2P_z, \quad n_x-n=B_1P_z, \tag{1}$$

P_z is the pressure in the z direction, n the index of refraction of the unstrained solid and n_z and n_x are the indices of refraction in the strained substance for light oscillating parallel and perpendicular to the direction of pressure. In most investigations only the difference

$$B=B_1-B_2=(n_z-n_x)/P_z \tag{2}$$

is determined.

We assume that the birefringence is caused by the elastic deformations[8] and consider therefore Neumann's strain-optical constants p and q as the fundamental quantities. They are defined by the equations

$$\begin{aligned} dn_z&=n_z-n=-n^2qz_z,\\ dn_x&=n_x-n=-n^2pz_z\\ n_z-n_x&=(p-q)n^2z_z, \end{aligned} \tag{3}$$

where dn_z and dn_x are the changes of the index of refraction caused by a strain z_z. p and q can be calculated from the experimental constants B_1 and B_2 by using the relations

$$\begin{aligned} p-q&=EB/n^2(1+\sigma),\\ 2p+q&=E(2B_1+B_2)/n^2(1-2\sigma), \end{aligned} \tag{4}$$

where E is Young's modulus and σ Poisson's ratio.

[1] H. Ambronn, Göttinger Nachrichten **2**, 299 (1919).
[2] O. Wiener, Ber. d. Ges. Wiss. Leipzig **62**, 255 (1909).
[3] W. Voigt, Ann. d. Physik **6**, 459 (1901).
[4] K. F. Herzfeld, J. Opt. Soc. Am. **17**, 26 (1928).
[5] K. Banerjee, Ind. J. Phys. **2**, 195 (1928).
[6] This will be shown in a paper on the photoelastic effect in cubic crystals.
[7] E. G. Coker and L. N. G. Filon, *Photoelasticity*, Cambridge Univ. Press, 1931, p. 199—use the notation $C_1=B_2$, $C_2=-B_1$, $C=B$. By choosing the above notation all quantities are usually positive.
[8] Filon and Jessop, Phil. Trans. **A223**, 91 (1922) find above the elastic limit proportionality with the stress but not with the strain. We consider only small deformations and assume Hooke's law to hold.

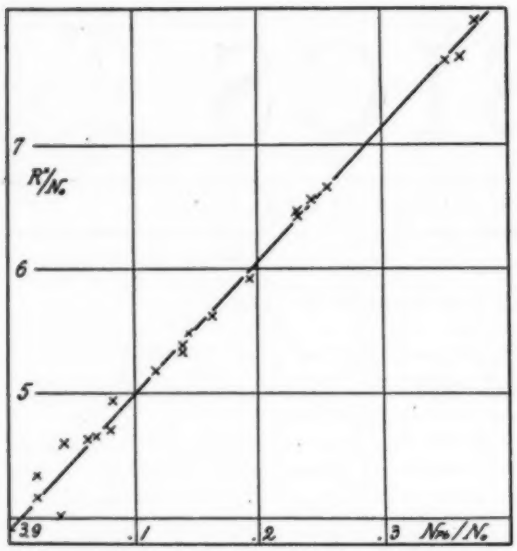

FIG. 1. Showing the validity of the Lorentz-Lorenz equation for lead-glasses.

If the solid is subjected to a hydrostatic pressure no double refraction appears, but the index of refraction changes by

$$dn=(q+2p)n^2d\rho/3\rho, \qquad (5)$$

where $d\rho$ is the change of the density ρ.

THE LORENTZ-LORENZ EQUATION FOR GLASSES

We assume the validity of the Lorentz-Lorenz equation

$$(n^2-1)/(n^2+2)=\rho\sum N_iR_i, \qquad (6)$$

where N_i is the number of gram equivalents of atoms of the type i contained in 1 g of the solid, and R_i their molar refraction. The derivation of this law assumes a random distribution of the atoms. For silica glasses Warren[9] has shown that each Si atom is tetrahedrally surrounded by four oxygen atoms, and each 0 atom is shared between two tetrahedral groups, but the relative orientation of two neighboring groups about their line of bonding is wholly random. In silicates and borosilicates over 90 percent of the total refraction is due to the 0 atoms and since the x-ray analysis gives a fairly random distribution of these atoms our assumption seems justified. (6) leads to reasonable values for the molar refraction of oxygen, namely, for the D line, $R_D=3.7$ in silicates, $R_D=3.4$ in borates and $R_D=3.9$ in lead glasses. This variation agrees with the rules of Fajans and Joos.[10]

Fig. 1 shows that the law (6) holds for lead glasses. The refraction of commercial glasses whose chemical composition is given in the I.C.T.[11] was calculated from the values of n_D and ρ given in Hovestadt's tables.[12] From the total refraction we subtracted the small refractions due to the cations Si, B, Al, K, Na, As, excepting Pb, and thus found R^*. The figure shows that $R^*/N_0=R_0+N_{Pb}\ R_{Pb}/N_0$ and gives $R_0=3.9$, $R_{Pb}=10.7$. The refraction of the bivalent Pb ion in PbO is naturally much larger than the value 1.56[13] of the four valent ion.

CHANGE OF THE POLARIZABILITY WITH PRESSURE

A serious objection against the validity of the Lorentz-Lorenz equation for glasses has been raised by Pockels.[14] From (6) follows

$$dn=(n^2-1)(n^2+2)d\rho/6n\rho. \qquad (7)$$

Pockels has shown (see Table I) that this equation gives an appreciably larger change of the index of refraction with density than is found experimentally from the photoelastic data, using (5). Since in deriving (7) only the change of the number of atoms has been taken into account, this discrepancy can best be explained by assuming that an increase of density decreases the polarizability of the atoms. This assumption is a generalization of the results of Fajans and Joos. It is well known that the refraction of ions decreases as the distance of the nearest neighbors gets smaller. No quantum-mechanical theory of this effect has as yet been given, but it seems logical that the transition probabilities and energy levels of the optical electrons will change if the atom is confined to a smaller space.[15]

[9] B. E. Warren, Phys. Rev. **45**, 657 (1934).

[10] Fajans and Joos, Zeits. f. Physik **23**, 1 (1924); C. P. Smyth, *Dielectric Constants and Molecular Structure*, p. 159.

[11] Int. Crit. Tables, Vol. II, p. 89.

[12] H. Hovestadt, *Jenaer Glas*, 1900.

[13] J. H. Van Vleck, *Electric and Magnetic Susceptibilities*, p. 225.

[14] F. Pockels, Ann. d. Physik **7**, 745 (1902).

[15] Spedding and Bear, Phys. Rev. **42**, 58, 76 (1932); **43**, 287 (1933); **46**, 308, 975 (1934).

TABLE I. *Polarization data on various glasses.*

I.N.	$(n-1)10^3$	$\frac{(n^2-1)(n^2+2)}{6n}$	$q+2p$	$(q+2p)n^2/3$	L	L_1	L_2	Observer
134	507.5	0.600	0.474	0.358	0.402	0.818	0.194	Pockels[18]
(138)	512.3	.608	.453	.345	.432	.872	.212	"
45	545.2	.656	.492	.392	.403	.823	.193	"
(0935)	550	.662	.357	.357	.461	.866	.259	Filon[19]
60	570	.695	.525	.432	.379	.779	.179	Pockels
(65)	573	.699	.587	.484	.307	.730	.093	Twymann[23] and Perry
100	644	.814	.568	.512	.371	.757	.178	Pockels
108	751	.997	.586	.600	.398	.766	.214	"
114	962	1.414	.673	.865	.389	.745	.211	"

For a small change ΔV of the atomic volume we assume therefore

$$R=R_i(1+\lambda\Delta V), \tag{8}$$

and if all atomic volumes change by an amount proportional to the total volume change one finds

$$dn=(n^2-1)(n^2+2)(1-L)d\rho/6n\rho, \tag{9}$$

where
$$L=\sum\lambda_i N_i R_i/\sum N_i R_i. \tag{10}$$

In Table I the values of L have been calculated for nine glasses for which all necessary data are known.

L has nearly the same value for all glasses although it appears to decrease somewhat with increasing concentration of PbO. Since for the crown glasses L is practically identical with λ for oxygen, we conclude that a decrease of the atomic volume of the oxygen ion of one percent changes its polarizability by about 0.4 percent. For the Pb ion this decrease is slightly less.

ANISOTROPY OF THE LORENTZ-LORENZ FORCES

In an amorphous solid the electric field acting on any atom is $F=E+(4\pi/3)P$, where E is the electric field of the light wave and $P=E(n^2-1)/4\pi$ is the optical polarization. This result is derived by considering a spherical boundary with the arbitrarily chosen atom in its center. The polarization of the atoms outside the boundary gives the field $(4\pi/3)P$ and the field of the dipoles within the sphere is

$$F_z^1=3\sum\mu_i z_i^2/r_i^5-\sum\mu_i/r_i^3. \tag{11}$$

For a random distribution of the dipole moments μ_i we have

$$\sum\mu_i x_i^2/r_i^5=\sum\mu_i y_i^2/r_i^5 =\sum\mu_i z_i^2/r_i^5=\tfrac{1}{3}\sum\mu_i/r_i^3, \tag{12}$$

and hence $F_x^1=F_y^1=F_z^1=0$.

A strain z_z produces a homogeneous deformation of the network of the atoms. The average distance between nearest neighbors in the direction of strain is larger than in the direction normal to the strain. Hence (12) is obviously no longer correct and it is easy to show that $F_x^1=F_y^1>0$, $F_z^1<0$. Hence the rearrangement of the atoms changes the Lorentz-Lorenz force, and this change depends on the orientation of E with respect to the direction of strain.

Havelock[16] suggested that in the deformed state we should consider a boundary having the form of an ellipsoid of rotation. If its axial ratio is $1:1:1+\epsilon$ the polarized medium outside the boundary gives the field

$$\begin{aligned} F_x&=E_x+4\pi P_x(1/3+2/15\epsilon),\\ F_y&=E_y+4\pi P_y(1/3+2/15\epsilon),\\ F_z&=E_z+4\pi P_z(1/3-4/15\epsilon). \end{aligned} \tag{13}$$

ϵ must be a small quantity. (13) gives the Lorentz-Lorenz force provided the eccentricity ϵ can be chosen so as to give $F_x^1=F_y^1=F_z^1=0$ where F^1 is the field due to the dipoles within the ellipsoid.

It is easy to prove that this condition is satisfied if $\epsilon=z_z$. In this case the ellipsoid contains the same atoms as are found within a sphere in the undeformed state. If x_i, y_i, z_i are the coordinates of an atom in the unstrained state, its coordinates in the deformed state are x_i, y_i, $z_i(1+z_z)$.

[16] T. H. Havelock, Proc. Roy. Soc. **A80**, 31 (1908).

Introducing these values in (11) and developing in powers of z_x gives

$$F_x^1 = 3z_x[\sum \mu_i z_i^2/r_i^5 - 5\sum \mu_i x_i^2 z_i^2/r_i^7],$$
$$F_y^1 = 3z_x[\sum \mu_i z_i^2/r_i^5 - 5\sum \mu_i y_i^2 z_i^2/r_i^7],$$
$$F_z^1 = 3z_x[3\sum \mu_i z_i^2/r_i^5 - 5\sum \mu_i z_i^4/r_i^7],$$

where the summations are to be carried out over the positions in the undeformed state. But for a sphere, assuming originally random distribution

$$3\int z^2/r^5 \cdot dV = 5\int z^4/r^7 \cdot dV = 15\int x^2 z^2/r^7 \cdot dV,$$

and hence $F_x^1 = F_y^1 = F_z^1 = 0$.

The indices of refraction are found from the equation

$$(n_x^2 - 1)/\rho = 3\sum N_i R_i[1 + 4\pi(\tfrac{1}{3} + (2/15)z_x)P_x/E_x],$$

and two analogous equations for n_y and n_z. Differentiating the solutions with respect to z_x and comparing the result with (3) gives

$$p = (n^2-1)(n^2+2)/6n^3 - (n^2-1)^2/15n^3,$$
$$q = (n^2-1)(n^2+2)/6n^3 + (n^2-1)^2 2/15n^3. \quad (14)$$

In the differentiation the R_i have been considered as constants.

The first term in both equations is due to the change of density, and leads, using (5), to Eq. (7). This term is mainly responsible for the observed fact that p and q and hence also B_1 and B_2 have always positive values. The second term gives the birefringence due to the anisotropy of the Lorentz-Lorenz forces

$$p - q = -(n^2-1)^2/5n^3, \quad (15)$$

(15) leads to values of $(p-q)$ of the correct order of magnitude, but the negative sign is inconsistent with all observations. According to (4) B has the same sign as $(p-q)$ and a negative value of B means that a pressure produces positive uniaxial birefringence. However, with the exception of two cases, all glasses and amorphous solids become negatively doubly refracting under pressure.

Anisotropic Polarizability of Atoms

A satisfactory theory can evidently only be given if one also takes into account the changes of the atomic refractions produced by a strain. In accordance with the previous consideration we must assume that the mechanical deformation of a solid deforms the ψ functions of the optical electrons and creates an anisotropy of the polarizability of the atoms. The linear character of the photoelastic effect leads to the assumptions

$$R_z = R(1+\lambda_1 z_z),$$
$$R_x = R_y = R(1+\lambda_2 z_z). \quad (16)$$

This gives a temporary double refraction due to the deformation of the atoms

$$n_z - n_x = L^* z_z (n^2+2)(n^2-1)/6n^3, \quad (17)$$

where

$$L^* = L_1 - L_2 = \sum(\lambda_1 - \lambda_2)_i N_i R_i / \sum N_i R_i, \quad (18)$$

and we have the relations with the previously introduced quantities λ and L

$$\lambda = (\lambda_1 + 2\lambda_2)/3, \quad L = (L_1 + 2L_2)/3. \quad (19)$$

Combining both effects (15) and (17) gives the final result

$$(p-q)n^3/(n^2-1) = EBn/(n^2-1)(1+\sigma)$$
$$= L^*(n^2+2)/6 - (n^2-1)/5. \quad (20)$$

Comparison with Experiments on Glasses

Determinations of the stress optical constant B have been made by Wertheim,[17] Pockels,[18] Filon,[19] Harris,[20] Jessop,[21] Heymans and Allis,[22] Twyman and Perry,[23] Savur,[24] and Adams and Williamson.[25] Harris found that the results vary with the age of the glass. Glasses of nearly the same composition but of different manufacture give occasionally very different results. Measurements on different pieces of the same

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[17] G. Wertheim, Ann. Chim. Phys. **40**, 156 (1854).
[18] F. Pockels, Ann. d. Physik **7**, 745 (1902); **9**, 220 (1902); *Lehrbuch der Kristalloptik*, Leipzig, 1905.
[19] L. N. G. Filon, Proc. Camb. Phil. Soc. **12**, 313 (1904); Proc. Roy. Soc. **A83**, 572 (1910).
[20] F. C. Harris, Proc. Roy. Soc. **A106**, 718 (1924).
[21] L. N. G. Filon and H. T. Jessop, Phil. Trans. **A223**, 89 (1922).
[22] P. Heymans and W. P. Allis, J. Math. Phys. **2**, 216 (1924).
[23] F. Twyman and J. W. Perry, Proc. Phys. Soc. **34**, 151 (1922).
[24] S. R. Savur, Phil. Mag. **50**, 453 (1925).
[25] L. H. Adams and E. D. Williamson, J. Wash. Acad. Sci. **9**, 609 (1919).

TABLE II. *Photoelastic data on glasses of approximately known composition.*

I.N.	$(n_D-1)10^3$	$\frac{E10^{-2}}{2(1+\sigma)}$	$B\times10^6$	$(p-q)10^3$	$\frac{(p-q)n^2}{(n^2-1)}10^2$	$L^*\cdot100$	$C_0 10^2$	$C_{Pb}10^2$	Ref.	Glass type
	458	30.9	3.33	97	26.7	71.5	99		22	Fused Quartz
		25			21.5	64.3	99		26	" "
(14)	500	32.9	3.17	93	24.1	69	98		19	3296
134	507.5	18.8	4.32	71	19.2	62.4	99		18	S205 Borate
	512.3	33.4	2.77	81	21.7	66	97		18	0428
19	516	32.8	2.85	81	21.9	66.7	97		25	Boro-Silicate
36	523	28	2.57	62	16.6	60	94		25	Crown
41	536.8	21.6	2.69	50	13.5	55.8	90	8	19	0152
41	536.8	21.6	3.55	66	17.7	61.2	90	8	24	1809
45	545.2	21.9	3.78	69	18.4	63	88	11	18	0658
(45)	545	21.9	3.37	62.5	16.5	60.6	88	10	21	3413
(45)	550	23.4	3.20	62	16.5	60.6	86	12	19	0935
60	570.0	25	2.92	60	15.7	60	82	14	18	2154
60	571	25	2.81	58	15.1	59.4	82	14	19	0154
(60)	572	25	2.25	46	12.1	55.6	82	14	20	2783
(65)	573	26.4	3.49	75	18.6	64.2	80	15	23	Light Flint
65	573	25	3.20	65	17.1	62.4	80	15	25	" "
67	574	29.5	2.81	67	17.7	63	82	13	25	" "
82	606	26	3.10	63	16.4	63	82	12	25	Ba-Flint
83	608	29	2.15	48	12.7	58.2	85		25	Ba-Crown
95	616	22.2	3.15	54	14.0	60	72	25	25	Med. Flint
96	620.3	22.2	2.79	48	12.7	58.2	72	24	19	0103
100	644	22.3	2.59	43	11.1	57.6	70	27	18	1571
(101)	650	22.3	2.24	37	9.6	55.8	70	27	24	HF6652
103	655	22.3	2.67	43.5	11.3	58.2	70	27	25	Heavy Flint
105	673	22.2	2.20	35	9.1	60.6	67	30	19	0192
107	717	22.2	1.75	26.5	6.8	55.8	64	34	19	041
(107)	740	22.2	1.35	20	5.1	54	62	36	24	4840
108	751	22.2	1.37	20	5.2	55.2	59	38	18	0500
110	756	21.8	1.22	17	4.5	54.6	58	40	25	Extra Heavy Flint
111	778		very small				58	41	19	0198
	880	21.2	−0.19	−2.3	−0.6	54.6	55	45	18	Special Glass
114	962.5	20	−1.89	−19.7	−5.2	53.4	49	51	18	S57
114	962	20	−0.95	−9.9	−2.6	55.8	49	51	19	S57

[21] By using the values of E and σ found by Schulze, Ann. d. Physik **14**, 384 (1904).

kind of glass made by the same or different observers sometimes vary considerably. Test of (20) requires the values of E, σ and n. In many cases these values are not known, but if the chemical composition is given they can be computed by interpolation and with the help of the formulas of Winkelmann and Straubel.[12]

In Table II we have collected all the data available on glasses of approximately known composition. B and n are for sodium light. n_D is in all cases sufficiently well known. The values of B and E are referred to a pressure of 1 kg per mm². The average error for B and E is at least five percent and an error of ten percent for $p-q$ is a very conservative estimate. The first column gives Morey's index number[11] I.N. A bracket indicates that the glass has not the same but a very similar composition as the one given in the I.C.T. $C_0=N_0R_0/\Sigma N_iR_i$ and $C_{Pb}=N_{Pb}R_{Pb}/\Sigma N_iR_i$ are estimated values. L^* is is calculated from Eq. (20). The last two columns contain the references and the manufacturers number of the glass. In Fig. 2 the values of $(p-q)n^3/(n^2-1)$ and L^* are plotted as functions of n^2. The most reliable data on glasses for which all constants have been measured, are represented by large circles. The less reliable values, for which E and σ were found by interpolation,

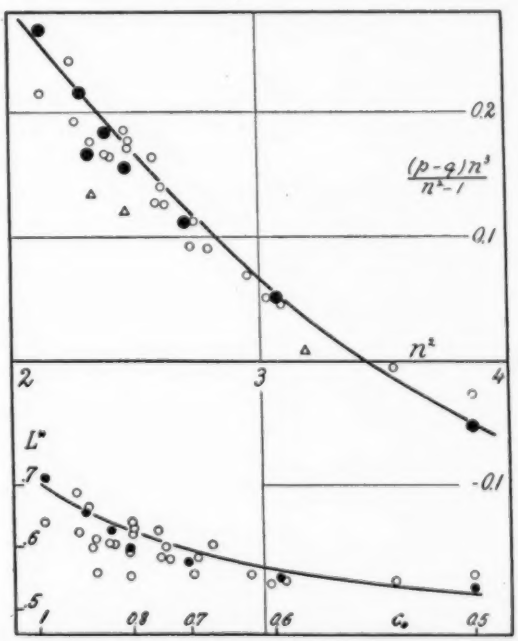

FIG. 2. Dependence of the photoelastic effect of glasses on the index of refraction.

are shown as small circles and the triangles indicate that the results seem questionable.

Making allowance for the large experimental uncertainties the figure demonstrates the dependence of the strain optical constant on the index of refraction. For glasses whose refraction is largely due to the oxygen ions the values of L^* are somewhat larger than for the lead glasses. The data can be represented by a formula of the type (18) $L^*=0.7C_0+0.35C_{Pb}$ or

$$L^*=0.35(C_0+1). \tag{21}$$

This means that for oxygen $\lambda_1-\lambda_2=0.7$ while for the positive ions this value is about one-half as large. Obviously the polarizability of positive ions, since they have lost the outermost electrons, is less affected by external pressure than the polarizability of the negative ions. From (19), using $\lambda=0.4$, we get for oxygen $\lambda_1=0.87$ and $\lambda_2=0.17$. This means that a reduction of the diameter of an oxygen ion by one percent, leaving the cross section normal to the strain unchanged, reduces its polarizability in the direction of strain by about 0.9 percent. In addition, it leads to the somewhat unexpected result that the polarizability perpendicular to the strain is also slightly decreased by about 0.15 percent. The values of L_1 and L_2 in Table I indicate that this is also true for the cations.

The curve in Fig. 2 is calculated from Eq. (20) taking into account the empirical law (21) and the variation of C_0 with n as indicated in Table II and in Fig. 2.

The numerical values of the constants λ which characterize the deformability of the atoms are approximate ones. We should expect that they differ for different cations. Also for oxygen they will not be the same in borate, silicate and lead glasses. But the data are not accurate enough to distinguish such differences.

The influence of age is probably due to crystallization. We shall show in another paper that in a lattice the anisotropy of the Lorentz-Lorenz forces produced by strains is usually smaller than in amorphous solids, and hence the crystallization process increases the photoelastic constants.

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Some Observations on the Movement and Demagnetization of Ferromagnetic Particles in Alternating Magnetic Fields[1]

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The movement of ferromagnetic particles in various types of alternating magnetic fields and the mechanisms involved are discussed. The conditions under which demagnetization takes place in magnetic fields produced by damped high frequency oscillatory discharges are described, and possible practical applications to the magnetic separation of ores and to routine laboratory magnetic testing of iron and steel are indicated.

WHILE studying the magnetic properties of ferromagnetic mineral particles and by watching their behavior to determine the best conditions for their separation in alternating magnetic fields,[3] certain incidental observations were made which are not only very interesting but also, in one case at least, have proved to be of considerable value, resulting in the discovery of a rapid, practical method for the demagnetization of all ferromagnetic substances.[4]

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[2] Metallurgical division, U. S. Bureau of Mines.

[3] R. S. Dean, V. H. Gottschalk and C. W. Davis, *Magnetic Separation of Minerals*, Report of Investigations 3223, U. S. Bureau of Mines, 1934, pp. 3–13; V. H. Gottschalk and C. W. Davis, *An Apparatus for Determining the Magnetic Constants of Mineral Powders*, Report of Investigations 3268, U. S. Bureau of Mines, 1935 p. 51; C. W. Davis, *Practical Aspects of A. C. Magnetic Separation*, Report of Investigations 3268, U. S. Bureau of Mines, 1935, p. 101; R. S. Dean and C. W. Davis, *Magnetic Concentration of Ores*, Trans. Am. Inst. Min. Met. Eng. **112**, 509 (1935).

[4] C. W. Davis, *A Rapid Practical Method of Demagnetization Involving High Frequency*, submitted for publication in Nature.

Behavior of Ferromagnetic Particles in Alternating Fields of Moderate Frequencies

Some information concerning the action of ferromagnetic particles in alternating magnetic fields of moderate frequencies (60–350 cycles per second) has been given by Mordey and others[5] who observed that certain ferromagnetic mineral particles vibrated in alternating magnetic fields and that the rate of motion increased with the frequency up to about 85 cycles and then decreased with increasing frequency, becoming zero at 150 to 350 cycles per second.

Further information is given in a paper published in February, 1934,[6] by Dean, Gottschalk and Davis. Reporting results of observations made about a year earlier, they state, with reference to the vibrations of particles resting on a surface in a field produced by single-phase alternating current:

> The mechanism of this activity in the alternating magnetic field is readily understood on the basis of high coercive force. The small permanent magnets are oriented by the first direction of the field, and when in the reversal they find themselves with their poles oriented against the field they are therefore repelled. Soft magnetic materials reverse their poles with the current and are not therefore repelled.

Hatfield[7] discusses this action at some length but makes no distinction between the behavior of particles of low and high coercive force in such alternating fields as contrasted to that in moving fields of alternating polarity.

Particle movement in single-phase alternating fields

The difference in action between particles of high and of low coercive force in single-phase alternating magnetic fields has been well-demonstrated by an apparatus developed for exhibition purposes. The magnetic equipment consists of two identical magnets actuated with alternating current; each comprises two coils of wire wound on cardboard cylindrical tubes of different diameters, and one coil is slipped within the other. The magnets are placed side by side, on end, and are connected in series so that when activated by a suitable alternating current, each produces a fairly uniform field of equal strength, not to exceed 200 oersteds at the inside surface of porcelain evaporating dishes resting on the tops of the magnets. Particles of low H_c, represented by magnetite, soft iron, or permalloy, placed in one dish become arranged in stationary radiating lines just as would be the case with a unidirectional magnetic field. The strength of the field exceeds the H_c of the material, so that the polarity rather than the orientation of the particles changes as the field reverses. Magnetized particles of high H_c, exemplified by partly reduced hematite, magnet steel, or heat-treated minerals, such as chromite or lepidocrocite, placed in the other dish act in a strikingly different way. They dance and bound about incessantly. The coercive force of the material exceeds the field strength, and the particles tend to reorient themselves at each reversal of the field.

Recent experiments furnish semiquantitative data on the action in the frequency range up to about 500.[8] It appears that there are three different ranges of field strength within each of which a different effect occurs. The dividing line between the zone of immobility and that of activity is determined, at low frequencies, by the magnetic properties of the material, particles with low remanence moving as soon as the field is strong enough to move a single particle, while particles with sufficient remanence to cause agglomeration move in chunks when the field is sufficiently strong. The upper limit of the range of field strength within which a ferromagnetic particle vibrates is determined by the coercive force of the particle, and when the coercive force is sufficiently exceeded the polarity of a particle rather than the particle itself alternates with the field. As the frequency increases the agglomerated particles with high remanence break up and the particles vibrate independently of each other.

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[5] W. M. Mordey, *The Concentration of Minerals by Means of Alternating Currents*, Min. Mag. (London) **28**, 333 (1922); B. W. Holman, *Recent Research in Ore Dressing*, So. African Min. Eng. J. **36**, pt. 2, 138, 171 (1925); J. A. L. Ortlepp, *Alternating Current in Magnetic Separation*. J. Chem. Met. Min. Soc. So. Africa **30**, 99 (1929).

[6] R. S. Dean, V. H. Gottschalk and C. W. Davis, reference 3, p. 13.

[7] H. S. Hatfield, *The Action of Alternating and Moving Magnetic Fields upon Particles of Magnetic Substances*, Proc. Phys. Soc. (London) **46**, (5) 604 (1934).

[8] R. S. Dean and C. W. Davis, reference 3; C. W. Davis' *Magnetic Properties of Mineral Powders and Their Significance*, Report of Investigations 3268, U. S. Bureau of Mines, 1935.

The range of field strength within which motion takes place appears to increase somewhat with the frequency up to a maximum and then gradually decreases, eventually to zero. The relation of frequency to the range of field strength in which particles are active is, for a given particle size, apparently specific for each specimen. Ordinary magnetite with a coercive force of 20 oersteds, sized between 150 and 250 mesh, vibrates within a very limited field-strength range, which becomes zero at about 160 cycles, but partly reduced hematite sized between 60 and 150 mesh with a coercive force of about 425 oersteds vibrates over a rather wide field-strength range and at frequencies in excess of 510 cycles per second.

Particle movement in moving, alternating fields

The action of particles in alternating moving fields may be quite different and although dependent on fixed directions of easiest magnetization is independent of the coercive force of the substance. An explanation for the action was evolved while studying a magnetic separator described by M. Ruthenburg[9] in which particles, resting on a flat surface held over a two-pole d.c. magnet revolving around a horizontal axis, moved over the surface in a direction opposite to that of the magnet pole below the surface. The action depends on the fact that, by virtue of their form or structure, practically all ferromagnetic particles possess one resultant direction of easy magnetization; in other words, each particle on magnetization becomes a magnet with poles at definite positions. Elongated, demagnetized particles and magnetized particles of low coercive force when placed in a unidirectional magnetic field assume polarity such that the nearer ends and the adjacent magnet pole will have opposite signs while magnetized particles with sufficient coercive force will take their position in accordance with their fixed polarity even if compelled to turn through an angle of 180°. The manner of particle movement as shown in the accompanying diagram (Fig. 1) is readily seen if the speed of magnet rotation be slow. The particles take positions in the field in accordance with their direction of easiest magnetization with their *N* ends to the right, and as the *N* pole of the magnet approaches below the surface from the left the particles gradually rise on their *S* ends with their *N* ends in the air; as the *N* pole of the magnet is directly below they stand upright and then as it passes the particles drop back to the surface with their *N* ends toward the left in proper position to become the lower ends during subsequent rotation as the *S* pole of the magnet passes underneath. Actually particles are held together in elongated bars because of induced magnetism, and the bars turn end over end toward the left. It is thus seen that regardless of whether or not the particles are permanent magnets their motion in a moving field of this type will be the same. Particles of soft iron, permalloy, magnetite, magnet steel and partly reduced hematite all move in the same way.

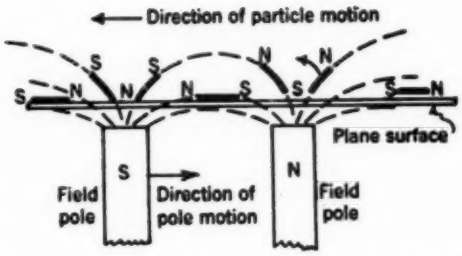

FIG. 1. Diagram showing the action of particles in a moving magnetic field.

BEHAVIOR AT HIGH FREQUENCIES

At high frequencies rather unexpected results were obtained. Information concerning the condition prevailing within and near the ends of a solenoid during the passage of damped high frequency oscillatory discharges through it was obtained by observing the behavior of various substances under the influence of the field at those points.

Particle movement in high frequency fields

A series of tests was made with a water-cooled solenoid 3 inches in diameter, 3½ inches high, and containing 6 turns per inch, connected to an Ajax-Northrup, 3-kva mercury-gap converter, the primary circuit of which was supplied with 5 amperes by a 220-volt, 60-cycle line. The peak

[9] M. Ruthenburg, U. S. patent 686,835, November 19, 1901.

amperage of the oscillatory discharges was not known and the frequency was estimated to be of the order of 50,000. Ferromagnetic particles placed in a container and held inside the solenoid were agitated violently. In contrast to the action at moderate frequencies the activity is apparently independent of coercive force, but is roughly proportional to the magnetic susceptibility of a substance. Thus particles of permalloy, soft iron and magnet steel with comparatively high susceptibility but with wide variation in coercive force, vibrate to approximately the same degree, particles up to 20 mesh jumping as high as 2 inches; Mineville magnetite with moderate susceptibility and low coercive force[10] is intermediate; and partly reduced hematite with lower susceptibility but with high coercive force[11] is least active. This behavior suggests that the polarity of a particle cannot alternate rapidly enough to keep in phase with the changing field so that occasionally the polarity, even in particies of low coercive force, is such that the furious onslaught of changing fields causes particle motion. It is also possible that the zone of maximum field intensity surges along the axis of the solenoid, thus causing particle movement. The electrical conductivity does not appear to have a marked effect on the action of particles but in the case of strips the effect is different. Granules of copper, brass and aluminum do not move, but strips of sheet iron, copper, brass or aluminum, when placed in a test tube and held inside the solenoid, take positions longitudinally parallel to the axis of the solenoid. It would appear that eddy currents in the metal cause an induced magnetic field around the nonmagnetic pieces.

Demagnetization by high frequency fields

The independence of particle activity and coercive force under these conditions is apparently related to the fact that previously-magnetized ferromagnetic substances have become demagnetized by the action within the solenoid. Qualitative tests indicate that practically complete demagnetization may be effected in this way. Lumps and small particles of lodestone magnetite or of strongly-magnetized, partly-reduced hematite with a coercive force of over 400 oersteds lose all tendency to stick together after demagnetization, while the remanence of cobalt steel magnets may be decreased to such an extent that permalloy powder is not supported at surfaces that were strong magnet poles before demagnetization. The most complete demagnetization is obtained if the specimen, which should be wholly within the solenoid, is placed so that a line between its poles is approximately parallel to the axis of the solenoid and if the current be switched off with the specimen in place. Sufficient demagnetization to prevent ore particles from sticking together may be accomplished by simply pouring the magnetized material through the solenoid as fast as possible.

A further study of the effect of high frequency damped oscillatory discharges was made by an arrangement which provided a means for regulating the number of discharges per unit of time, and permitted current control, so as to eliminate power consumption in excess of that required for the demagnetization of the sample under investigation. This was accomplished by causing interruptions in a direct current passing through the solenoid windings by means of a rotary switch, across the gap of which a condenser had been connected as shown diagrammatically in Fig. 2. For most purposes a solenoid consisting of about 5000 turns of No. 22 double-cotton-covered magnet wire wound over a 3-inch length of $1\frac{3}{8}$-inch diameter, nonmetallic tubing was found to be satisfactory when a voltage of 110 was applied to it. The capacity of the condenser is apparently not very critical, a capacity of 4 microfarads giving a satisfactory balance of this solenoid, but it should be such that a spark instead of an arc occurs on breaking the circuit. A more careful study of this point should be made for quantitative work. Material placed within the solenoid becomes demagnetized by the damped oscillatory discharge resulting from a single break in the circuit. In this case also the most complete demagnetization may be produced by placing the specimen so that it is well within the solenoid and is favorably oriented; that is, the specimen should be in the strongest part of the field and should have the same rela-

[10] C. W. Davis, *Magnetic Properties and Orientation of Ferromagnetic Particles*, Physics **6**, 96 (1935).
[11] V. H. Gottschalk and C. W. Davis, *A Magnetic Material of High Coercive Force*, Nature **132**, 513 (1933).

tive orientation in the demagnetizing field that obtained in the magnetizing field. Another precaution that must be taken in operating this type of demagnetizer to ensure satisfactory results is to break the circuit very rapidly. This may be accomplished by a sharp hammer blow on the handle of a knife switch. A snap switch has proved to be unsatisfactory. Occasionally an unusually loud discharge takes place as the circuit is broken and then the demagnetization is less complete. If magnetic measurements are to be made at low values of induction the further precaution of placing the solenoid with its axis at right angles to the earth's field may be justified. For continuous operation, such as would be required for the practical demagnetization of minerals, the speed of the rotary switch should be so regulated that material dropped through the solenoid becomes subjected to the field resulting from at least one complete damped oscillatory discharge. Demagnetization in this case, although not complete for unfavorably oriented particles, is sufficient to remove all tendency for attraction between particles.

Although, strangely enough, this method is exactly opposed to the, for this purpose almost useless, recommended procedure for demagnetization in which slow current reversals and a slowly decreasing field are employed,[12] it is particularly adapted to the demagnetization of finely divided particles of relatively high coercive force such as the previously-mentioned, partly-reduced hematite. The widely-used method of gradually decreasing the field strength of alternating magnetic fields of moderate frequencies is also unsatisfactory for the demagnetization of minerals with high coercive force, and undamped oscillatory high frequency alternating fields are apparently worthless for demagnetization.[13]

Partial demagnetization through the action of damped oscillatory discharges was observed by Henry and subsequently studied by Raleigh,

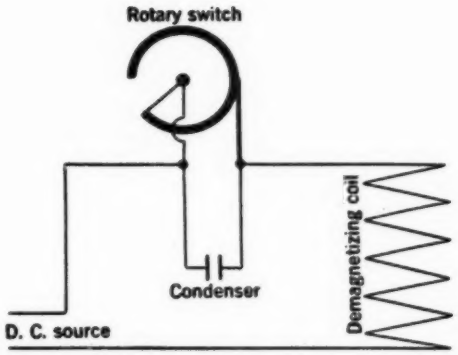

FIG. 2. Schematic diagram of demagnetizing coil.

Rutherford, Marconi and Tschetverikowa,[14] but none of these investigators seem to have recognized in the action a practical method for demagnetization. The field strengths used were, in most cases, too low for complete demagnetization even though the materials studied had comparatively low coercive force.

As a tentative explanation of the effectiveness of high frequency in this connection the author has suggested[4] that the rate of magnetic field reversal is greater than that to which the poles of the magnetic substance can conform, thus permitting the particles to be caught in a more favorable position for demagnetization than if time were permitted for their orientation in the prevailing magnetic field, and that the fleeting high-peak amperage of the damped oscillations and internal shocks caused by the effect of high frequency discharge may also be contributing factors. The similarity between the destruction of the hysteresis of thin wires of iron or steel by mechanical disturbances and by magnetic oscillations has been recognized by Starling.[15]

Practical aspects of demagnetization in the production of superior magnetic concentrates

This method has already provided a means for the removal of permanent magnetism from iron

[12] C. W. Burrows, *On the Best Method of Demagnetizing Iron in Magnetic Testing*, Bull. 78, U. S. Bureau of Standards, 1908, pp. 205–274.

[13] S. K. Mitra, *On the Demagnetization of Iron by Electromagnetic Oscillations*, J. de physique **4**, 259 (1923); Comptes rendus **176**, 1214 (1923); S. Procopiu and N. Florescu, *Demagnetization of Iron and Nickel by High Frequency Alternating Fields*, J. de phys. et radium **4**, 251 (1933).

[14] R. Glazebrook, *A Dictionary of Applied Physics, London* **2**, 1922, p. 1040; E. Rutherford, *A Magnetic Detector of Electrical Waves and Some of Its Applications*, Proc. Roy. Soc. **A60**, 184 (1896); Phil. Trans. **A189**, 1 (1897); G. Marconi, *Magnetic Detector of Electric Waves*, Proc. Roy. Soc. **A60**, 341 (1902); M. Tschetverikowa, *Demagnetization of Iron Compounds by Electric Oscillations*, Zeits. f. Physik **44**, 139 (1927).

[15] S. G. Starling, *Electricity and Magnetism*. 4th ed., 1924, p. 445.

minerals that did not respond to usual demagnetizing methods so that the classification and the removal of waste matter from products of this character obtained during d.c. magnetic separation are now for the first time possible.

Another adaptation of the method which may use present equipment to effect a superior concentration of iron ores, although it may be found preferable to design new magnets with laminated cores, consists in using the rotary switch with its shunted condenser in the power line which energizes d.c. magnetic separators. Small-scale tests show that, with the switch operating slowly, magnetic material supported below a pole of such a magnet is cyclically picked up, held, demagnetized, and dropped so that a pile of such particles undergoing this action resembles a miniature volcanic eruption. Removal of part of the entrained nonmagnetic particles may be accomplished under these conditions, but if the speed of the rotary switch be properly adjusted the particles are picked up, then momentarily demagnetized and allowed to drop a short distance in this condition, liberating entangled nonmagnetic waste matter which continues to fall while the magnetic particles are again pulled back toward the magnet pole. Since this action may be repeated as long as desired an exceedingly complete separation should be possible.

Possible application of high frequency demagnetization to magnetic testing

The method has been of considerable assistance in demagnetizing mineral powders while investigating their magnetic properties. Further work outside the scope of our present investigations should be done to determine whether this method would be satisfactory for the routine laboratory testing of bars and rods of iron and steel. If so, it should be especially valuable for the demagnetization of the new magnet steels, such as K.S., M.K., Köster, and new K.S., with coercive forces of 500 to 900 or more oersteds.[16] It would also be interesting to learn whether the 18- to 24-hour period between demagnetization and testing, which Sanford[17] has found necessary for obtaining consistent results in the determination of permeability of ferromagnetic materials at low values of induction, might not be eliminated by demagnetization by means of fields produced by damped, high frequency, oscillatory discharges.

For demagnetizing rods and bars the solenoid should be somewhat longer than the test specimen and should have ample ampere-turns for the purpose. Toroids could perhaps be demagnetized by rotating them inside a solenoid, with the axis of rotation perpendicular to the plane of the toroid and to the axis of the solenoid, during several series of damped oscillatory discharges.

Spontaneous remagnetization is said to result when material has been demagnetized by a.c. magnetic fields[12, 13] but no such action was apparent, during a period of several weeks, in lumps of partly-reduced hematite and in rods of cobalt magnet steel with square cross section $\frac{3}{8}$-inch on a side and 3 inches long, that had been magnetized in a strong d.c. field and then demagnetized as described above.

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[16] K. Honda, H. Masumoto and Y. Shirakawa, *A New K. S. Permanent Magnet*, Science Reports, Tohoku Imperial University **23**, No. 3, 365 (1934).

[17] R. L. Sanford, *Drift of Magnetic Permeability at Low Inductions After Demagnetization*, Bur. Standards J. Research **13**, 371 (1934).

The Determination of Arc Temperature from Sound Velocity Measurements. I

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The dependence of the velocity of sound on gas temtemperatures is utilized to determine the temperature of the copper arc in air at atmospheric pressure. The sound "wave" produced from a condensed spark discharge in air is passed through the stabilized arc column and received by a second spark discharge, acting as a microphone. The time interval is measured on the cathode-ray oscillograph. When the arc length and, hence, sound path, are varied, the time interval varies approximately linearly. The resultant sound velocity free from end corrections is 1.39×10^5 cm sec.$^{-1}$, for which the arc temperature is calculated to be 4100 ± 300°K when corrections for dissociation and changes in specific heat are taken into account. Time intervals are measured for a constant arc length of 9 cm for currents between 3 and 26 amperes, and it is found that the velocity (and temperature) is independent of current within the experimental error. Conclusions based on the results are: (1) The electron density is 0.6×10^{12}; (2), the partial pressure of Cu vapor is 7.5×10^{-7}; and (3), the "effective ionization potential" of the arc gas is 12.3 volts.

IT was suggested[1] as early as 1873 that the dependence of the velocity of sound on the temperature of the gaseous medium might be employed as the principle of a pyrometer. The idea has not had a wide application, however, and the special advantages of the method in the determination of the very high temperatures of high pressure arc discharges do not seem to have been recognized. The measurement of the temperature of a gas from observations on the velocity of propagation of sound is on fundamental grounds a direct method since a sound wave moves through a gas as a small amplitude component superimposed upon the velocity of thermal motion of the molecules.

The velocity of sound c is related to the mean molecular velocity $\bar{n}$ by a constant, as expressed by

$$c=\bar{n}(\gamma\pi/8)^{\frac{1}{2}},$$

where γ is the ratio of specific heats. The factor $(\gamma\pi/8)^{\frac{1}{2}}$ takes into account (1) the difference between mean and r.m.s. values, (2) that c is directed while $\bar{n}$ is random, and (3) that the sound pressure changes are adiabatic.

The concept of the temperature of a gaseous discharge is in general not free from ambiguity, since the neutral molecules, the positive and negative ions, the atoms, and the free electrons, may have a Maxwellian velocity distribution of a different mean value at the same time in the same discharge tube. It is clear, however, that as the pressure in the tube is raised the temperature of the lighter components, as, for example, the electrons, will exceed the temperature of the neutral gas molecules by a smaller and smaller amount, and they will finally become identical. There has recently appeared[2, 3] direct experimental evidence that at a pressure of one atmosphere the electron temperatures in arcs in air are equal to the gas temperatures. The same must be true at higher pressure and is certainly true at somewhat lower pressures.

Temperature Dependence of Velocity of Transmission; Approximate Theory

The velocity of transmission c of a small amplitude pressure wave through an elastic medium at the absolute temperature T_0 is given by $(K/\rho_0)^{\frac{1}{2}}$, where K is the volume modulus of elasticity and ρ_0 is the density at the temperature T_0. For adiabatic pressure changes,

$$K=\rho_0 dp/d\rho=\gamma p_0, \tag{1}$$

hence

$$C^2=\gamma p_0/\rho_0, \tag{2}$$

where p_0 is the pressure at T_0 and γ is the ratio of specific heats. The ratio p_0/ρ_0 is independent of the pressure in the case in point, and γ is a constant for a given gas, so that c is independent of the pressure.

The constant γ is independent of the temperature, provided the specific heats of the gas

[1] Mayer, Phil. Mag. **45**, 18 (1873).

[2] Mannkopff, Zeits. f. Physik **76**, 396 (1932); **86**, 161 (1933).

[3] Witte, Zeits. f. Physik **88**, 415 (1934).

do not change, but ρ_0 varies inversely as the absolute temperature, so that $c \sim T^{\frac{1}{2}}$. That is, the velocity of transmission of a sound wave through a gas varies directly as the square root of the absolute temperature. The velocity c is independent of density changes, due to pressure at constant temperature, depends upon density changes due to temperature, and also depends on density changes due to changes in composition, nature of the gas, or dissociation.

This simplified treatment of sound velocity applies to many practical cases. For the special case of a dissociating, ionizing gas at high temperature, however, account must be taken of the change in density due to dissociation, as well as a temperature dependence of γ. This "constant" γ is properly defined in terms of the specific heats of a gas, quantities which in turn have a temperature dependence at very high and very low temperatures. If the appropriate ρ and γ are known at any given temperature, the corresponding sound velocity can be calculated.

The details of this calculation are given in a paper by H. Poritsky and C. G. Suits which immediately follows this one. An experimental method of measuring the sound velocities in arcs is described in the present paper.

Abnormal apparent sound velocities are observed[4, 5] near the source of intense sounds, but these high velocities attenuate to 331 m sec.$^{-1}$ (at 0°C) within very small distances from the point of origin. For the sound source used in this experiment the normal velocities, in air at room temperatures, are observed for all distances greater than one centimeter.

Experimental Method

The method employed for determining the velocity of sound in arcs at atmospheric pressure is shown schematically in Fig. 1, where the arc column F burns between the cathode A and anode B.

The sound is produced directly in the discharge column by means of a condensed discharge between the tungsten probe C and the arc cathode A. The gap between C and A is shunted by the arc and a resistor of 10,000 ohms so that the breakdown is determined by an external series sphere gap H. A very intense sound wave is produced directly in the arc by this method.

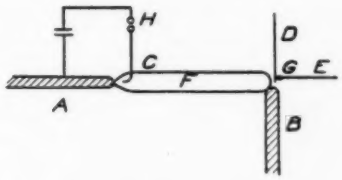

Fig. 1. Schematic diagram of apparatus.

The condenser energy used for the sound gap is of the order of 150 coulombs at 14 kv. More important than a high energy input is a high discharge rate; in the present experiment the natural frequency of the discharge circuit was usually 75 kc. Since the wave travels more rapidly in the high temperature core, the sound energy attenuates very rapidly because of the increased convexity of the wave front. If the energy input is too low, or the wave front insufficiently steep, reception will be limited to short arcs, or will be absent.

The Sound Receiver System

We have found that the passage of a sound "wave" through a low current, high voltage, nonoscillatory spark discharge results in a sudden change in spark voltage. The spark voltage change due to a pressure wave is proportional to the spark length; there is a directional effect and a small polarity effect, such that the best response is had when the sound wave is traveling in the direction of the spark axis from cathode to anode. This microphonic property of spark discharges is employed as shown in Fig. 1, where the spark G passes between the electrodes D and E. The changes in spark voltage are sufficiently large to allow them to be recorded on the high voltage cathode-ray oscillograph without amplification. The properties of this spark receiver are in many ways quite ideal for the present experiment, since the high time resolution allows the intervals of 30 to 150 microseconds for sound passage in arcs of convenient length to be recorded with satisfactory accuracy; and the fact that the receiver can tolerate a high

[4] Mach and Gruss, Wien Ber. **78**, Part 2, 467 (1879).
[5] Payman, Robinson and Shepherd, Safety of Mines Research Board Papers 18 and 29, 1926.

ambient temperature makes it possible to place it very near the arc and thus minimize end zone errors. The spark receiver cannot, however, be placed directly in the arc column, whereas the sound spark can be so placed. It is necessary to separate the receiver spark from the arc under investigation by $1\frac{1}{2}$ cm.

The property of sparks of detecting pressure waves,[6] as well as the reaction of an arc to these waves, are matters of much interest which may form the subject of a subsequent paper.

Circuits and Auxiliaries

A detailed sketch of the experimental set-up and oscillograph circuits is shown in Fig. 2. The arc chamber, somewhat more elaborate than indicated in Fig. 1, employs two water-cooled electrodes A, B, the latter of which is hollow and holds the receiving spark G, made of 0.01″ wires, with a gap of 8 mm. The sound electrode C is a 0.10″ W rod providing a gap of 5 mm to the cathode. The sound gap is energized by the condenser C_1 of 1.5 mf at 14 kv, which is synchronized with the sweep circuit by ultraviolet light from the gap J. The breakdown of the gap J is timed by the magnitude of C_2 and R_2. An improved method of timing the oscillograms is used in this experiment. To avoid errors due to variations in deflection sensitivity with position on the screen and changes in the degree of vacuum,—errors which may accumulate to 15 percent—the timing oscillation is included in the sweep circuit by means of the coupling coil T.

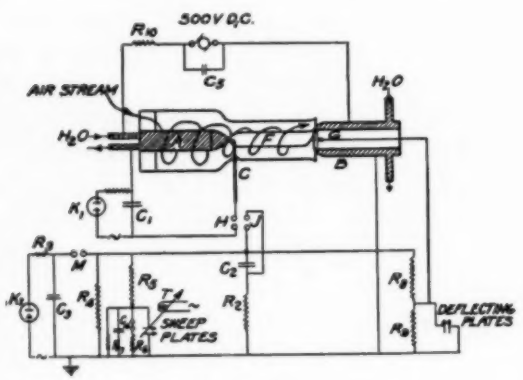

FIG. 2. Arc electrodes and oscillograph circuits.

The motion of the cathode beam across the screen takes place with a constant velocity upon which is superimposed a small sinusoidal component of velocity which oscillates in the direction of motion of the beam. The timing frequency is such that from 50 to 100 oscillations take place in the width of the screen. For small deflections, the appearance of the line on the screen is beaded or dotted, the dot period being one or more microseconds. In the present experiment this timing method has greatly improved the accuracy.

Stabilizing the Arc

The arc is stabilized[7, 8] with a turbine jet arrangement providing a circulating air stream in which the air describes a corkscrew path in a glass tube. This excellent method of stabilizing allows the use of long straight discharge columns.

Experimental Results

In Fig. 3 is shown schematically the travel of the cathode-ray beam, most of which occurs off the screen together with the portion photographed. Voltage is applied to the receiving spark G almost instantly after the 70 kv cathode gap M breaks down. The voltage of the receiving gap G falls very rapidly to a small value, but the discharge across the gap persists for approximately 1000 microseconds as a non-oscillatory spark discharge. The constants C_2 and R_2 are so adjusted that the trigger gap J breaks down about 100 microseconds after the receiving gap G. Ultraviolet light from gap J causes the gap H, which is overvoltaged, to break down. The oscillatory discharge of condenser C_1 produces a single steep wave front which travels toward the receiving spark. When the pressure front reaches the receiving spark, the spark voltage rises steeply. The time of initiation of the sound wave is marked on the film as a damped oscillation, which is an electrostatic pick-up from the sound circuit C_1-H-C. This is shown in Fig. 3 as the point t_0. The subsequent rise in spark voltage is shown at the point t_1. The time interval between t_0 and t_1 is the recorded time of passage.

[6] Not a fundamentally new property, apparently. See Thomas, Trans. A. I. E. E. **42**, 1111 (1923).

[7] Lotz, Zeits. f. tech. Physik **5**, 187 (1934).
[8] Traub, Ann. d. Physik **18**, 169 (1933).

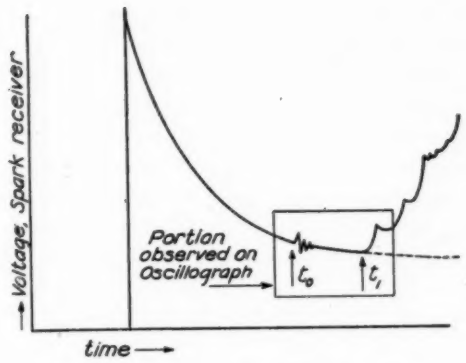

FIG. 3. Schematic oscillograph record.

The experimental method is tested by measuring sound velocity at room temperatures. When the time intervals are plotted as a function of distances (between 1 and 12 cm), the result is a straight line with a slope agreeing, within the experimental error of about 3 percent, with accepted values. By extrapolation to zero, it is found that abnormally high velocities, characteristic of explosions, exist and attenuate to normal values within one cm from the source. This observation is in agreement with Foley,[9] who found very rapid attenuation of explosion velocities from sparks in air. Because of the diffraction of sound energy to the cooler boundary layers, it is likely that abnormal velocities attenuate still more rapidly in the arc.

Oscillograms taken for air at room temperatures at two different values of sound path are shown in Fig. 4a, where several superimposed traces appear on each film.

A procedure similar to that described above is used when the temperature determination is made on an arc. The arc is started by touching a Cu starting rod to the cathode and drawing the discharge over to the anode. Oscillograms taken in the Cu arc appear in Fig. 4b. Data obtained in this way are shown in Fig. 5, where the arc current was maintained at a constant value of 6 amperes for different values of arc length between 4 and 14 cm. The slope of this curve is the velocity of sound free from the corrections due to the cold end zone. These data suggest

[9] Foley, Phys. Rev. **35**, 373 (1912); **16**, 449 (1920); Proc. Nat. Acad. Sci. **6**, 310 (1920).

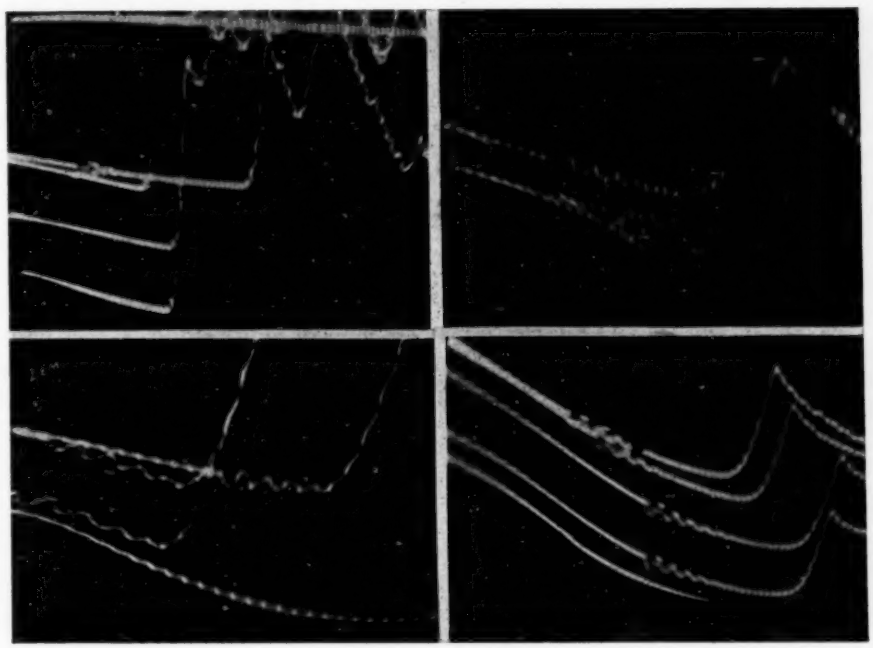

FIG. 4a. Oscillograms taken for air at room temperatures at two different values of sound path. 118.5 kc dotted sweep.

FIG. 4b. Oscillograms taken for the copper arc, two values of sound path. (Fig. 4b should be reversed, the top section being made the bottom, and *vice versa*.)

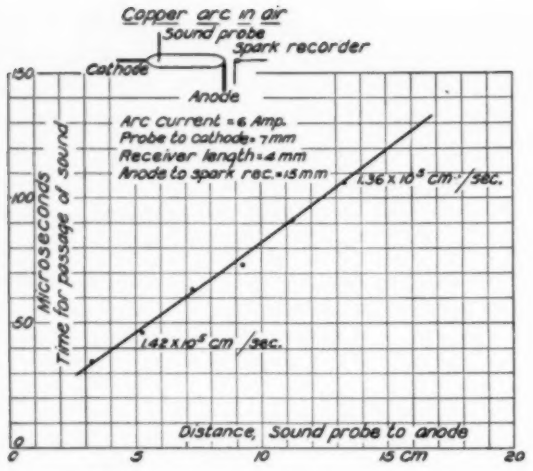

FIG. 5.

some slight curvature in the time *vs.* distance plot which may be interpreted as a slightly higher temperature for the shorter arcs. The maximum velocity for the short arcs is 1.42×10^5 cm/sec., while the minimum velocity, found for the long arcs, is 1.36×10^5 cm/sec. The corresponding temperatures, not corrected for the effect of dissociation, are 5000°K and 4600°K, respectively. When the dissociation and changes in the specific heat are taken into account, the corrected values of temperature are $4200°\pm200°$K and $4000°\pm200°$K, with an average value of $4100°\pm300°$K. When the arc length is maintained at a constant value of 9 cm and the arc current is varied between 4 and 26 amperes, it is found that the sound velocity is independent of the current within the spread of the measurements. The consequences of this constant temperature are of considerable theoretical interest and will be discussed in a separate paper. This measurement is in good agreement with $T=(4600\pm350)°$K, obtained by von Engel and Steenbeck[10] for the copper arc under nearly identical experimental conditions.

Discussion of Sources of Experimental Error

Lack of perfect time resolution in the spark receiver leads to a spread in observations of the time intervals. This appears in an uncertainty in the determination of the point t_1 on the oscillograms. In the data presented, the values for t_1 have been read by determining the point at which the spark voltage first rises.

The exact point on the curve which is chosen for t_1 is of no consequence, providing the same point is selected in all oscillograms for the different arc lengths. An actual error in the time intervals, due to the smaller amounts of energy received for the greater arc lengths, is of more serious consequence. This uncertainty may, however, be tested experimentally by varying the sound energy and observing the time intervals for constant arc length. These experiments show that it is practical to use sufficient sound intensity to avoid these errors due to rapid attenuation of energy with distance. The fact that the observed temperatures are independent of arc current over a sixfold range of current bears on this point, since there is a very great difference in the amount of sound energy received for a 4-ampere arc and a 26-ampere arc. This effect, if present, is therefore small. Thermal diffusion is a possible source of error which, if corrected for, would lead to a higher temperature. Unfortunately, few of the coefficients which would allow an accurate calculation of the magnitude of the effect are available for these experimental conditions. Thermal diffusion and concentration diffusion are further discussed in the following paper.

Discussion of Results

From the observed temperature the kinetic theory mobility k can be calculated from the Langevin formula or its refinements. The density of ionization, N_I, can then be calculated from the known arc gradient E and current density I from the relation

$$I=N_I\,ev, \quad \begin{matrix} e=\text{electronic charge} \\ v=\text{electron velocity} \end{matrix} \tag{3}$$

where

$$v=kE. \tag{4}$$

The arc gradient E for the 6-ampere copper arc is obtained from the slope of the straight line portion of the arc voltage-arc length characteristic and is found to be 16.5 volts cm^{-1}. The current density is not an entirely satisfactory quantity to measure in these arcs since the boundary of the core of the discharge is not too

[10] von Engel and Steenbeck, Wiss. Veroff. a.d. Siemens-Konzern **10**, 155 (1931); **12**, 74, 89 (1933).

well defined. However, within the uncertainty imposed by that factor, the current density of the stabilized atmospheric pressure Cu arc is a constant and is equal to 10 amperes cm^{-2}.

The quantity N_I obtained in this way is 0.6×10^{12} cm^{-3} which for x, the fraction ionized, yields the value 0.3×10^{-6}.

The density of ionization N_{II} can be obtained from the Saha equation in the form

$$\log_{10} N^2_{II}/N_0 = -5040\,V_i/T + 3/2\log_{10} T + 15.385. \quad (5)$$

In the above equation N_0 is the density of molecules at the absolute temperature T, and V_i is the critical potential, in this case the ionizing potential. From observations on the spectrum we know that the column of this arc is characterized by a strong N_2 band spectrum, upon which are superimposed some Cu lines. We may assume, therefore, that the arc gas is N_2 with a small partial pressure of Cu vapor. We find that the partial pressure of Cu vapor required to bring N_{II} into agreement with N_I is 7.5×10^{-7} atmosphere; only three percent of the charge density is contributed by the nitrogen, the other 97 percent is provided by the ionized copper vapor.

It should be pointed out that a factor of 2 or 3 times in the calculation of mobility will not greatly influence this conclusion, since this difference, interpreted in temperature, is small.

From the relatively great effect of a small amount of Cu vapor it is clear that those factors which influence the amount of Cu vapor in the arc, as, for example, the condition of the electrodes,[11] are uncontrolled variables in the case of temperature determinations on metallic arcs in air. This variable factor may be expected to lead to somewhat different temperatures in the case of the same arc studied by different observers, each with his own technique for preparation of the electrode surface. This variable factor must also lead to some temperature fluctuations and, hence, spread in the measurements, since the amount of metallic vapor in the arc changes from time to time as observed from the variations in spectral quality of the light. This is particularly true in the case of arcs employing anodes of low boiling point metals.

The copper electrode tips used for the temperature determination reported here were well oxidized; it is likely that higher temperatures would be obtained from tips of highly polished copper, resurfaced after each experimental point is taken, since the positive column of the arc between polished copper electrodes is blue in color and nearly free from copper vapor.

An alternative interpretation of the data can be made by calculating an "effective ionizing potential," V_{ie}, in the following manner. For the observed temperature it is assumed that a homogeneous gas with an ionizing potential V_{ie} provides the charge density N_I calculated from the current density, gradient and mobility. Thus for

$$N_0 = 1.82\times10^{18}\ cm^{-3},$$
$$N_I = 0.6\times10^{12}\ cm^{-3},$$
$$T = 4000°K,$$

one obtains from (5)

$$V_{ie} = 12.3 \text{ volts},$$

which is in good agreement with $V_{ie}=11$ volts obtained by von Engel and Steenbeck[10] for similar experimental conditions. This interpretation amounts to a "calibration" of Eq. (5), which can then be used in further calculations not too far removed from the calibration point. Whether or not (5) without calibration will precisely predict results which agree with experiment, for ideal experimental conditions, when and if found, remains somewhat questionable. It is hoped that this point will be clarified by the future results of an experiment now in progress.

[11] Suits, Physics **5**, 380 (1934).

The Determination of Arc Temperature from Sound Velocity Measurements. II

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Experimentally determined values of the velocity c of sound in atmospheric pressure arcs in air are interpreted in terms of the temperature of the gas. The velocity may be expressed in general in the form:

$$c=((RT/M)(1+R/C_v))^{\frac{1}{2}},$$

which applies to pure gases or mixtures of gases that obey the perfect gas laws. In this relation T is the absolute temperature; M is the mean molecular weight defined as: $M=\Sigma M_i p_i/\Sigma p_i$; C_v is the specific heat (per gram molecule) defined as: $C_v=\Sigma(C_v)_i p_i/\Sigma p_i$, where the summation is taken for all the gas components present. In the case of the arc in air, the components considered are the undissociated and dissociated oxygen and nitrogen. The values for the specific heat and degree of dissociation, taken from the equilibrium calculations of H. L. Johnston and associates are used, care being taken to apply the correct partial pressures. Possible sources of error due to thermal diffusion and other causes are considered.

1. Sound Velocity in Gaseous Mixtures

CONSIDER a homogeneous mixture of several gaseous compounds and let f_1, f_2, $\cdots$ be the fraction of the molecules of the first, second, $\cdots$ components or constituents in that mixture, so that $\sum f_1=1$. Define the molecular weight M of the mixture as the mean molecular weight $\bar{M}$ of all the molecules:

$$M=\bar{M}=\sum M_i f_i, \tag{1}$$

where M_i is the molecular weight of the ith component. M grams of the mixture will be referred to as a "gram mole." Thus defined, a gram mole of the mixture, just as in the case of pure compounds, will contain a total of $N=6.062\times10^{23}$ ($N=$ Loschmidt's number) molecules, and

$$N_i=f_i N$$

molecules of the ith constituent.[1] We denote by v the volume of such a gram mole. This volume v is a function of the pressure p and absolute temperature T given by the equation of state:

$$v=v(p, T). \tag{2}$$

While in (2) the independent variables are p and T, the equation of state could, of course, be formulated by expressing p as a function of v and T, or, again, as a function of the density $\rho=M/v$ and T. If the two degrees of freedom allowed by the equation of state are reduced to one by making p and ρ (or p and v) interdependent, then the velocity c of propagation of small disturbances from an initial uniform state is given by the well-known relation:[2]

$$c^2=dp/d\rho,$$

or, replacing ρ by M/v, by

$$c^2=-(v^2/M)dp/dv. \tag{3}$$

The velocity c of sound propagation is obtained from (3) by making the pressure changes adiabatic, thus determining the ratio of dp/dv. This results in the expression:

$$c^2=-(v^2/M)\frac{1+(p/C_v)(\partial v/\partial T)_p}{(\partial v/\partial p)_T}$$

$$=-(v^2/M)(\partial p/\partial v)_T\gamma, \tag{4}$$

where C_v, C_p are the specific heats in ergs per gram mole, and $\gamma=C_p/C_v$.

To calculate the velocity of sound of the mixture from properties of the constituents by means of (4), it will suffice to express the equation of state (2) and the specific heat C_v of the mixture in terms of those of the components. For "non-reactive" mixtures, this may be done as follows:

$$v=\bar{v}=\sum v_i(p, T)f_i, \tag{5}$$

$$C_v=\bar{C}_v=\sum(C_v)_i f_i, \tag{6}$$

where v_i, $(C_v)_i$ refer to the volumes and specific

[1] This is really the sense in which the conventional chemical molecular weights and gram moles of the elements are to be interpreted in the light of their isotopic constitution.

[2] Rayleigh, *Theory of Sound*, Vol. 2, p. 15.

heats per gram mole of the ith constituent. "Nonreactive" is used in the following sense: Suppose the proper amount of each gaseous component is brought to the pressure p and temperature T in sealed containers separated by thin partitions. If these partitions are broken while the temperature of the common container is held at T, the mixing will take place without change of pressure while the heat intake will be equal to the external work that can be obtained from the diffusion of the molecules through semi-permeable membranes. For such nonreactive mixtures we consequently have:

$$c^2=-(\bar{v}^2/M)\frac{1+(p/C_v)(\partial\bar{v}/\partial T)_p}{(\partial\bar{v}/\partial p)_T}. \qquad (7)$$

For mixtures that suffer volume changes Δv, or changes in internal energy E upon mixing (under constant pressure and temperature), $\bar{v}$ and $\bar{C}_v$ should be replaced by $\bar{v}+\Delta v$, $C_v+E/T+p(\partial\Delta v/\partial T)$, respectively.

For perfect gases the universal equation of state holds,

$$pv=RT, \qquad (8)$$

where $R=8.316\times10^7$ ergs/degree$=1.988$ cal./degree$=Nk$, k being the Boltzmann constant:

$$k=1.373\times10^{-16}\text{ erg/degree}.$$

For mixtures of (chemically nonreactive) perfect gases, elementary kinetic theory considerations show that the mixture is nonreactive in the sense of obeying Eqs. (5) and (6). Consequently the mixture, too, obeys the law of perfect gases, and Eq. (4) reduces to:

$$c^2=(RT/M)(1+R/C_v)=(RT/M)\gamma, \qquad (9)$$

while (7) becomes:

$$c^2=(RT/M)(1+R/C_v). \qquad (10)$$

In evaluating $\overline{M}$, $\overline{C}_v$, for perfect gases, the weights may be taken equal to the partial pressures p_i, thus

$$\overline{M}=\sum p_iM_i/\sum p_i.$$

While (10) constitutes the most convenient form for the sound velocity for gaseous mixtures in arcs because specific heat curves are available for many gases at high temperatures,[3] direct determination of γ for the mixture is at times desirable. Since

$$\gamma=R/C_v+1, \quad C_v/R=1/(\gamma-1),$$

and C_v for the mixture is obtained by taking the weighted mean of $(C_v)_i$, it follows that while γ is *not* the weighted mean of γ_i, $1/(\gamma-1)$ *is* the weighted mean of $1/(\gamma_i-1)$. Thus

$$1/(\gamma-1)=\overline{1/(\gamma_i-1)}=[p_1/(\gamma_1-1)+p_2/(\gamma_2-1)+\cdots]/(p_1+p_2+\cdots).$$

In particular, for two gases, the above can be put in the form

$$\gamma=\frac{p_1\gamma_1(\gamma_2-1)+p_2\gamma_2(\gamma_1-1)}{p_1(\gamma_2-1)+p_2(\gamma_1-1)},$$

which agrees with the result of G. Schweikert.[4]

2. Dissociation of Air at High Temperatures —Application to Arcs

To calculate the velocity of sound in air at high temperatures, it is necessary to know the composition in order to determine the mean molecular weight $\overline{M}$ and specific heat $\overline{C}_v$. We shall take account of the dissociation processes

$$O_2\rightleftarrows 2O \qquad (11)$$

$$N_2\rightleftarrows 2N, \qquad (12)$$

neglecting the effects of diffusion, ionization, and the presence of other gases (such as water vapor, H_2, and noble gases).

It should be pointed out that in calculating the equilibria of polyatomic gases at temperatures as high as 5000° the usual forms of the equilibrium equation* do not apply. This is because of the variation of the specific heat with temperature due to the energy taken up by atomic and electronic vibrations. A similar situation occurs at low temperatures where the specific heat varies because of the energy in the rotational states. The "vapor pressure" constant[5] takes account of the integrated effect of this variation

[3] Johnston and Long, J. Chem. Phys. **2**, 389 (1934). Footnote 1.
[4] Schweikert, Ann. d. Physik **48**, 651 (1915).
* Saha and Srivastava, *A Textbook of Heat*, p. 457.
[5] Stern and Fowler, Rev. Mod. Phys. **4**, 365 (1932).

in specific heat and yields an equation applicable to the range of "ordinary" temperatures at which rotational equipartition is fully developed but below the temperatures at which the vibrational levels start absorbing energy.

Recently W. F. Giauque,[6] H. L. Johnston[7] and their associates have published the equilibrium constants

$$K_1=(p_N)^2/p_{N_2},\quad K_2=(p_O)^2/p_{O_2} \qquad (13)$$

of the reactions (11) and (12) for temperatures up to 5000°K. These constants are for pressures expressed in atmospheres and were calculated from specific heats and dissociation potentials of O_2 and N_2 determined from spectroscopic data. These constants and the specific heats (in calories per gram mole) are given in Table I.

TABLE I. *Equilibrium constants and specific heats of* N_2 *and* O_2.

Reference	(1)	(2)	(2)	(3)
T°K	K_1	C_p of N_2	K_2	C_p of O_2
3000	3×10^{-7}	8.86	1.42×10^{-2}	9.11
3500	2×10^{-5}	8.93	0.268	9.22
4000	7×10^{-4}	8.99	2.45	9.33
4500	1×10^{-2}	9.04	13.8	9.44
5000	8×10^{-2}	9.08	55.4	9.55

At normal temperatures air contains 78.05 percent O_2 and 20 percent N_2 by weight, so that

$$p_{O_2} : p_{N_2}=78.05/28 : 20/32=3.72 : 1.$$

Neglecting diffusion, this will remain the ratio of the total number of atoms of N to those of O, whether dissociated or undissociated:

$$2p_{N_2}+p_N : 2p_{O_2}+p_O=3.72 : 1. \qquad (14)$$

If x_1, x_2 represent the fractions of N_2, O_2 molecules that are dissociated, then

$$p_{N_2} : p_N=1-x_1 : 2x_1,$$
$$p_{O_2} : p_O=1-x_2 : 2x_1.$$

Hence,

$$p_{N_2} : p_N : p_{O_2} : p_O=3.72(1-x_1) : 3.72(2x_1) : (1-x_2) : 2x_2,$$

and, neglecting the partial pressures of other gases,

$$p_{N_2}+p_N+p_{O_2}+p_O=1=4.72+3.72x_1+x_2.$$

Solving for the partial pressures and substituting in (13), one obtains:

$$K_1=3.72(4x_1^2)/(1-x_1)(4.72+3.72x_1+x_2), \qquad (15)$$

$$K_2=4x_2^2/(1-x_2)(4.72+3.72x_1+x_2). \qquad (16)$$

Physically, these simultaneous equations for x_1, x_2 arise from the fact that the dissociation of either O_2 or N_2 affects the fraction of the pressure available for the other gas and its dissociated form. The Eqs. (15), (16) were solved by a highly convergent method of successive approximation. The results recorded in the second and third columns of Table II were obtained in this way. In the next four columns of Table II, the partial pressures p_{N_2}, p_N, p_{O_2}, p_O, calculated from x_1 and x_2, are tabulated. The last three columns contain data on $\overline{M}$, $\overline{C}_v/R$, and the velocity of sound c in cm per sec. The partial pressures p_{N_2}, p_N, p_{O_2}, p_O are plotted as a function of temperature in Fig. 1. It is interesting to note that the atomic oxygen has a maximum at 5000° and

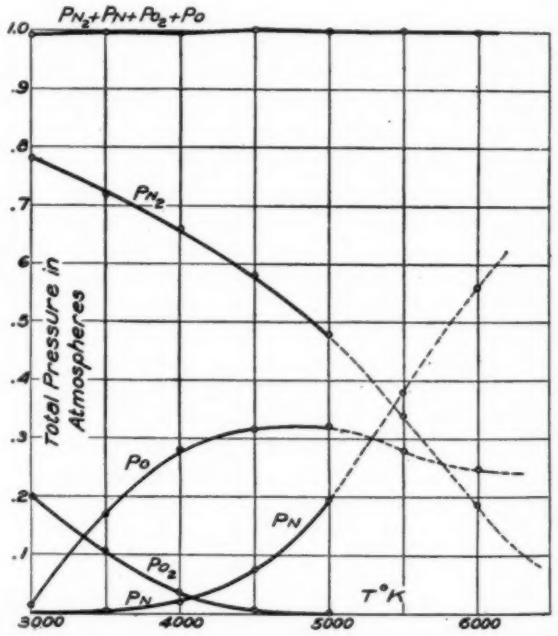

FIG. 1. Partial pressures of molecular and atomic oxygen and nitrogen in air between 3000°K and 6000°K. (Dotted portion from extrapolated equilibrium constants.)

[6] Giauque and Clayton, J. Am. Chem. Soc. **55**, 4887 (1933).

[7] Johnston and Walker, J. Am. Chem. Soc. **55**, 179, 191 (1933).

TABLE II. *Partial pressure data.*

T	x_1	x_2	p_{N_2}	p_N	p_{O_2}	p_O	$\bar{M}$	$\bar{C}_v/R$	c
3000	3.09×10^{-4}	0.1226	0.767	0.0005	0.182	0.0506	28.15	3.42	1.072×10^5
3500	2.65×10^{-3}	.4412	.717	.0038	.1081	.1707	26.32	3.16	$1.207 \times$ "
4000	1.608×10^{-2}	.790	.657	.0214	.0377	.2835	24.44	2.91	$1.350 \times$ "
4500	6.14×10^{-2}	.949	.593	.077	.0087	.322	23.10	2.69	$1.50 \times$ "
5000	1.680×10^{-1}	.988	.489	.1975	.0019	.3120	21.57	2.51	$1.64 \times$ "
*5500	.308	~1.0	.34	.38	.00	.28	19.32	2.35	$1.84 \times$ "
*6000	.592	~1.0	.19	.56	.00	.25	17.15	2.23	$2.06 \times$ "

* Based on extrapolated values of equilibrium constants.

for higher temperatures approaches the value 0.2 asymptotically.

The velocity of sound c is plotted as a function of the temperature T°K in Fig. 2, where the portion shown between 5000° and 6000° has been calculated from extrapolated values of the equilibrium constants. This curve was used in the evaluation of experimentally determined sound velocities in the copper arc (described in the preceding paper, I, of this journal) in terms of the gas temperatures.

3. Diffusion Effects in the Arc

The concentrations of O, N, O_2, N_2 were calculated in Section 2 on the assumption of thermodynamic equilibrium and on the assumption that the atmospheric pressure was divided between the nitrogen and oxygen, whether in dissociated form or not, in the same ratio within the core as outside (Eq. (14)). We now proceed to correct for these assumptions by taking into account the effects of *diffusion*.

The theory of molecular transport phenomena in a mixture of two gases was developed by Chapman[8] and Enskog.[9] The equation of interdiffusion for two gases (following Chapman's notation) is given by:

$$u_1\lambda_1 = -u_2\lambda_2 = -D_{12}(\partial\lambda_1/\partial x) - D_T(\partial T/T\partial x),^{10} \quad (17)$$

where:

λ_1, λ_2 are the fractional molecular concentrations of the two gases, 1 and 2; (these are identical with f_1, f_2 in the notation of the preceding sections);

u_1, u_2 are the mean molecular velocities of the two gases (drift velocities);

D_{12}, D_T are "coefficients" presently to be described.

The first term on the right side of (17) is due to concentration gradient and the coefficient D_{12} is "coefficient of diffusion" (really of "concentration diffusion"); the second term is due to temperature gradient and represents a flow known as "thermal diffusion," the coefficient D_T being known as the "coefficient of thermal diffusion." The values of D_{12}, D_T derived by theory depend upon the nature of the molecular forces and, in general, upon T and the concentration λ_1; they have been investigated for elastic sphere models and for various power laws of repulsion between molecules. The coefficient D_{12} is positive so that concentration diffusion tends to equalize concentration; the coefficient D_T is, in general, positive when the gas 1 is the heavier one, so that thermal diffusion tends to drive the heavier gas into the cooler region.

It follows from the above that, because of thermal diffusion, the heavier oxygen molecules

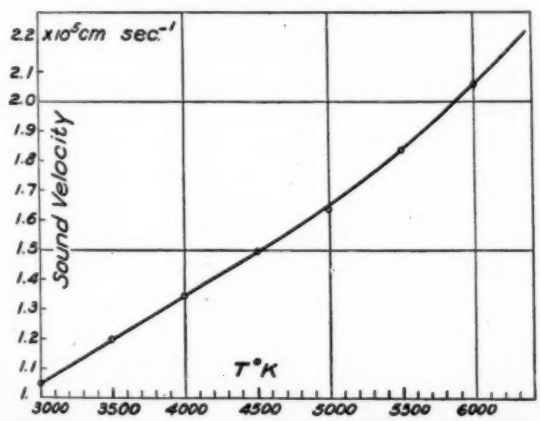

FIG. 2. Sound velocity as f (T°K). (Data of 1/22/35.)

[8] S. Chapman, Phil. Trans. Roy. Soc. **A217**, 115 (1916–17).

[9] D. Enskog, Inaugural Dissertation, Upsala, 1917.

[10] S. Chapman, reference 8, p. 138, Eqs. (10), (11); we have omitted terms due to pressure gradient and due to external forces and have further assumed zero net molecular flow.

and atoms will be driven from the arc and the nitrogen molecules and atoms towards the arc, until an equilibrium is reached wherein thermal diffusion is exactly opposed by concentration diffusion. Putting $u_1=u_2=0$ in (17), we find:

$$\partial\lambda_1/\partial x=-\partial\lambda_2/\partial x=-k_T\partial T/T\partial x, \quad (18)$$

where $k_T=D_T/D_{12}$, and integrating (18) on the assumption of constant k_T, we obtain:

$$\Delta\lambda_1=-k_T\ln(T_1/T_0). \quad (19)$$

This equation has been utilized for determining k_T experimentally;[11] the values so obtained lie below those predicted on the basis of elastic sphere models of molecules but agree with values such as would be expected on the basis of proper power laws of repulsion (greater than fifth) between molecules. We shall apply it for estimating a correction upon Eq. (14).

For O_2, N_2 mixtures at $p_{N_2}:p_{O_2}=3.72:1$, the elastic sphere model yields $k_T=0.009$.[12] With an estimated value of $k_T=0.005$ applied to both atomic and molecular mixtures of oxygen and nitrogen, we obtain from Eq. (14) for $T_1/T_0=5000°/300°$ a variation of concentration between the arc core and the surrounding air of at most 1.5 percent. This value should probably be increased, perhaps doubled, because of the atomic gases and the higher oxygen concentration in the core due to dissociation. With a 3 percent increase in nitrogen concentration in the core at 5000°K over that of surrounding air, the velocity of sound would exceed that of Table II by less than one percent.

We consider next the diffusion of the monatomic gases. Since their concentration vanishes outside the arc, they will tend to diffuse outward because of the concentration gradient, while the diatomic gases will diffuse inward. This motion will be opposed by thermal diffusion which now tends to move the lighter dissociated gases inward and the heavier ones outward. The magnitude of this thermal diffusion, estimated from available values of D_T, shows that the decrease in total molecular transport due to this effect may be as great as 50 percent. Even so, however, there results within the arc a decrease in p_O, p_N below the equilibrium values, this decrease being determined by the rapidity with which the diatomic O_2, N_2 in excess of equilibrium concentrations tend to dissociate.

To estimate the latter, suppose that a fraction $1/n$ of collisions between pairs of O atoms results in their combining into an O_2 molecule. The mean life of an O atom is then equal to the time required to cover n mean free paths corresponding to the partial pressure p_O. Applying equilibrium conditions with partial pressures p_{O_2}, p_O, we see that

$$\begin{pmatrix}\text{mean life of}\\ O_2\text{ molecules}\end{pmatrix}:\begin{pmatrix}\text{mean life of}\\ \text{O atoms}\end{pmatrix}=p_{O_2}:p_O$$

is less than 1 for $T>$about 3700°K, thus leading to a much shorter life for the molecular oxygen than for the atomic oxygen at the arc temperatures.

If we assume that the transition of temperature from that of the core to that of the outside takes place in a sheath of thickness 1 cm and that it is accompanied by a corresponding decrease in content of dissociated gases, we obtain from (17) an average drift velocity of the O atoms outward due to concentration diffusion:

$$u_1=-(D_{12}/\lambda_1)(\Delta\lambda_1/\Delta x)=-D_{12},$$

since λ_1 decreases to 0 within a distance $\Delta x=1$ and a similar drift of the O_2 molecules inward. With an estimated value of $D_{12}=25$ corresponding to the mean temperature 2500°K, and neglecting the opposing effect of thermal diffusion, the time it takes to cross the transition region is of the order of magnitude of 0.04 second.

While no great certainty exists in regard to the proper values of n, Hartel and Polanyi[13] have shown that in some cases $n=1$. In other cases existing theory postulates that an activation of the reacting molecules or atoms is necessary previous to collision in order that they interact.[14] Such activation presumably consists in their being brought to some higher quantum levels—a condition that is automatically helped by the high temperatures. On this basis we are led to a

[11] G. Blüh and O. Blüh, Zeits. f. Physik **90**, 12 (1934). This paper contains a complete bibliography of the literature on thermal diffusion.

[12] Chapman and Hainsworth, Phil. Mag. **48**, 604 (1924).

[13] Hartel and Polanyi, Zeits. f. physik. Chemie **B11**, 97 (1931).

[14] R. C. Tolman, *Statistical Mechanics*, Chap. 21.

low value of n, and consequently to a negligible correction for x_1, x_2 corresponding to the vanishingly small ratio of the mean lives of O or O_2 to 0.04 second.

4. Effect of Variable Velocity, Finite Amplitude and Time of Relaxation

As the sound wave penetrates the arc core it enters from a region of low temperature, and consequently low velocity, into a region of high temperature and high velocity. What effect has the passage through the boundary region in which the velocity varies rapidly with position?

The equation of propagation in a region in which the velocity c varies with position is

$$\partial^2\phi/\partial t^2=\nabla\cdot(c^2\nabla\phi),$$

but the integration is difficult for the case at hand, so that it was necessary to resort to graphical and experimental methods. The experiments were based upon the analogy between the propagation of sound waves in a gas, and "tidal" waves of a liquid, since the velocity of the latter varies with the depth.[15] This fact was utilized in a water tank of variable depth through which the passage of surface waves was observed. The experiment shows in a very simple manner that there is a rapid attenuation in wave amplitude as the wave front enters the region of greater depth and higher velocity.

The same result may be found by graphical plots of the propagation of a plane wave front through a region of variable temperature, T_0, T_1, T_2 $\cdots$ as shown in Fig. 3.

The successive wave fronts A, A, A, $\cdots$ are the envelopes of the Huygens construction for a convenient time interval t and are constructed so that the disturbance propagating along the normal rays B, B, B, $\cdots$ gets from one wave front to the next one in time Δt. If it is supposed that the energy travels normally to the wave front along these rays, it follows that the increased curvature of the wave front as it enters the high velocity regions is accompanied by a thinning-out of energy due to the divergence of the rays, and this can be quantitatively estimated from the diagram if backward reflection is neglected.

[15] Lamb, *Hydrodynamics*, 5th ed., p. 193, Eq. (2).

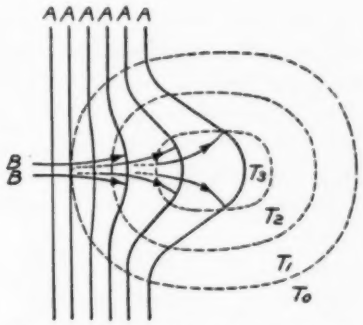

Fig. 3. Propagation of wave fronts.

It follows that high sensitivity of the sound detectors is needed to insure the detection of the arrival of the thinned-out wave front. The above attenuation was of direct interest in counterbalancing the effects because of the *finite amplitude* of the wave front.

It will be recalled that the conventional sound velocity refers to the propagation of *small* disturbances of pressure and density. Disturbances of large amplitude in general propagate with higher velocity and with change of shape, tending to accentuate the steepness of the compressive wave front until it becomes a wave of shock or impact.[16] The propagation of the latter involves a definite energy loss that goes into heating the gas over which the wave has passed, and this results in an automatic decrease of amplitude and velocity.

Obviously the increase of velocity due to finite amplitude, if not allowed for, will result in overestimating the temperature. Fortunately, as discussed above, the excessive curvature of the wave as it enters the hotter region rapidly decreases the amplitude and brings the velocity to its normal value within the core even in cases where in air excessive velocities occur. For the sound wave used in the arc experiments described in I, the velocity in air is known to be normal. For data since taken the region of abnormal velocity has been used because of some experimental advantages; the results will appear in a subsequent publication.

Added to the above diminution of amplitude is the damping due to *time of relaxation*. It was first pointed out by Einstein[17] that sound prop-

[16] Lamb, reference 15, pp. 281–284; *Handbuch der Physik* Vol. 7, p. 322.

[17] A. Einstein, Sitzb. der Akad. der Wien, Berlin **85**, 380 (1920).

agated in dissociated gases suffers absorption and exhibits dispersion phenomena because of the lag in establishing dissociation equilibrium in the sound wave. Similar effects due to the inertia or time of relaxation are exhibited by the molecules in taking up vibrational energy (of atoms or electrons) and result in absorption and dispersion even at room temperatures.

These phenomena have been investigated by Kneser[18] and Knudsen.[19] Both Einstein and Kneser confine their considerations to disturbances that involve time sinusoidally. The differential equation that applies to any type of disturbance is not difficult to set up and turns out to be similar to that obtained when viscosity is taken into account.[20] An alternative procedure, however, is to apply the Fourier integral representation to an arbitrary pulse thus breaking it up into sinusoidal components, each one propagating with its own velocity and damping. By investigating the Fourier integral it can be shown that the major part of the pulse energy lies in frequencies higher than the reciprocal of the time of relaxation; (the latter is of the order of magnitude of 1/1000 sec. or less.)[18] Thus the major part of the pulse energy is concentrated in frequencies that are not affected by the phenomenon under consideration, and the latter would be expected to influence only the steepness of the front without greatly modifying the group velocity.

Note added in proof: Professor Johnston has revised the thermodynamic properties of O_2 due to the inclusion of a $^1\Delta$ level in the O_2 molecule, and has kindly communicated his results to us. We find the effect of these changes on the calculated velocity of sound to be negligibly small. Of more consequence is the revision of heat of dissociation of N_2 by Herzberg and Sponer (Zeits. f. physik. Chemie **26**, Abb. **B1–2**, 1–7 (1934) to 7.34 electron volts (in place of 7.90 employed in the references cited in the above paper.

[18] H. O. Kneser, J. Acous. Soc. Am. **5**, 122 (1934).
[19] V. O. Knudsen, J. Acous. Soc. Am. **5**, 112 (1934).
[20] See, for instance, Lamb, reference 15, p. 612.

JUNE, 1935 PHYSICS VOLUME 6

The Temperature Variation of the Photoelectric Emission from Thorium Oxide

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(Received January 12, 1935)

The photoelectric emission from a platinum filament coated with thorium oxide seems to be due chiefly to that part of the spectrum lying between 2500A and 3100A. An increase in the photoelectric emission from the thorium-oxide-coated platinum filament was observed with increasing temperature from 23°C up to about 200°C, and a decrease from the same filament with increasing temperature from 200°C to about 400°C. A definite photoelectric emission was observed from this filament whether the corresponding thermionic current was saturated or not. No photoelectric fatigue was observed.

THE electron emission from an illuminated surface is generally thought of as having two components: One, a true thermionic emission which is in agreement with Richardson's law, and which is produced by an actual increase in the temperature of the photoactive surface when it is illuminated; the other, a surface photoelectric effect that obeys Einstein's equation and which is not responsible for its existence to any increase in the temperature of the photoactive surface, rather, it is a true light effect.

In this paper, the term photoelectric current will be used to mean the difference between the electron emission given off by a surface with and without illumination. Finally, an effort will be made to show that the observed photoelectric effect is largely a light phenomenon.

In 1921, Case[1] and Merritt,[2] each working with different composite surfaces, reported a large increase in the photoelectric emission from these surfaces with the temperature. The work of Arnold and Ives,[3] which immediately followed,

[1] T. W. Case, Phys. Rev. **17**, 398 (1921).
[2] E. Merritt, Phys. Rev. **17**, 525 (1921).
[3] Arnold and Ives, Proc. Nat. Acad. Sci. **7**, 323 (1921).

proved conclusively that there was a light effect along with a heat effect, and further, that the light effect was much larger than the heat effect corresponding to a given amount of incident light energy.

Koppius,[4] Gibbs and Meachem,[5] and Majorana[6] carried out similar experiments with varying results. The works of Henriot and Moens,[7] Crew,[8] Newbury,[9] Newbury and Lemery,[10] Berger[11] and Ramadanoff[12] are in qualitative agreement that there is an increase in the photoelectric current with the temperature for certain temperature ranges.

The first real qualification of this fact came when in 1929 Bodemann[13] made the observation that at potentials large enough to produce saturation of the thermionic current, the photoelectric current attained its smallest values. This statement is not in agreement with my observations on thorium oxide.

Apparatus

(1) Photoelectric cell

The photoelectric cell used is shown in Fig. 1. It consisted of a platinum strip A, 15 mm long, 4 mm wide and 0.002 mm thick, which served as a filament; and an anode (plate) B, which was a piece of 35-mil nickel wire bent in rectangular form so that it was practically of the same form as the outer edge of the filament. The plate was mounted with its plane parallel to the plane of the filament.

Both the plate and the filament were spot welded to individual 35-mil tungsten wires C, which served as supports and lead-in wires. These lead-in wires were brought into the cell through a pressed Pyrex-to-tungsten seal.

The body of the cell was made of Pyrex glass. A disk D, of clear fused quartz, which was sealed to the body of the cell by a graded-glass seal, served as a window.

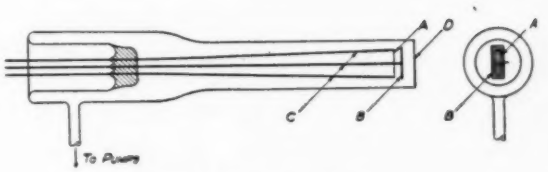

FIG. 1. Photoelectric cell.

The filament A was coated with thorium-oxide by putting a small quantity of an aqueous solution of thorium-nitrate on the thoroughly cleaned platinum filament and reducing it to the oxide by heating with an electric current.

The filament of the photo-cell was calibrated for temperature in terms of the filament current between 787°C and 1086°C by means of an optical pyrometer, which had been previously calibrated by means of a standard platinum-platinum rhodium thermocouple. Temperatures below 787°C were taken from the straight line drawn through the calibration points and passing through 23°C, at which temperature the filament current was zero. The various filament temperatures as determined by interpolation from this straight line are therefore only fair approximations.

The photoelectric cell was outgassed by baking for 70 consecutive hours in an electric furnace at 550°C. During this period of outgassing the filament was maintained almost constantly at 1086°C, this being a higher temperature than was used in any of the measurements that are reported here. The cell was considered thoroughly outgassed when there was no change that could be observed in the pressure within the cell with a large change in filament temperature.

(2) Vacuum system

The large bore Pyrex vacuum system was thoroughly torched before the cell was sealed off. One of the two liquid air traps for mercury was filled with charcoal granules and sealed off with the cell. Pressures of not more than 10^{-6} mm were obtained.

(3) Light source

The light source was a Gallois, fused-quartz, mercury arc operating on direct current (3.3 amperes at 110 volts). The complete radiation from this arc was passed through a diaphragm and a water-filled, clear, fused-quartz bulb,

[4] O. Koppius, Phys. Rev. **18**, 443 (1921).
[5] Gibbs and Meachem, Phys. Rev. **19**, 415 (1922).
[6] Majorana, Il Nuovo Cimento, p. 218 (1926).
[7] Henriot and Moens, Comptes rendus **180**, 651 (1925).
[8] W. H. Crew, Phys. Rev. **28**, 1265 (1926).
[9] K. Newbury, Phys. Rev. **34**, 1418 (1929).
[10] Newbury and Lemery, J. Opt. Soc. Am. **21**, 276 (1931).
[11] C. E. Berger, Phys. Rev. **34**, 1566 (1929).
[12] D. Ramadanoff, Phys. Rev. **39**, 884 (1931).
[13] E. Bodemann, Ann. d. Physik **3**, 614 (1929).

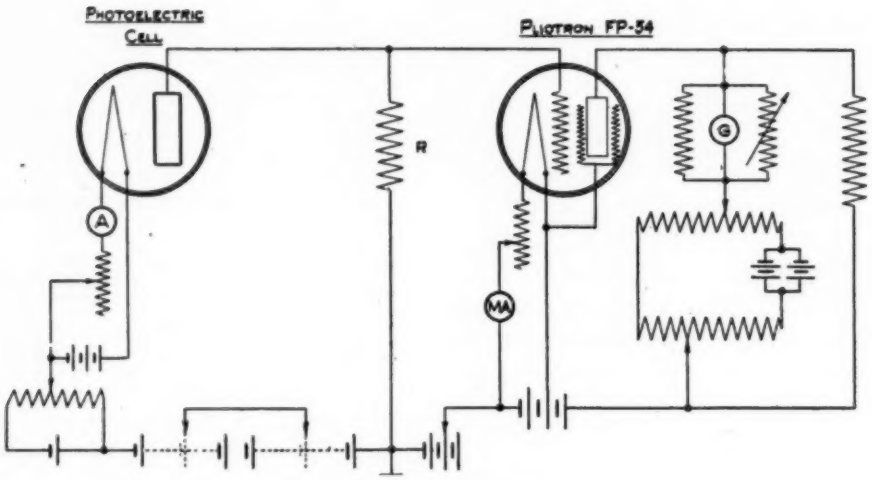

FIG. 2. Diagram of connections.

which was used for a condensing lens. By means of the water-filled lens, the radiation from the arc was focused (approximately) on the oxide-coated filament.

In all cases, the incident light was passed through this water-filled, quartz condensing lens, which was cooled by means of the air stream from an electric fan. The entire system was enclosed in a metal shield, which, in turn, was separated from the light source by suitable screens.

METHOD OF MEASUREMENTS

The single-tube pliotron circuit shown in Fig. 2 was used for making the electrical measurements.

The plate current of the plioton (G.E. FP–54) was measured by means of a D'Arsonval galvanometer having a current-sensitivity of 3×10^{-10} ampere/mm.

Every precaution was taken to prevent electrical leakage. For example, all apparatus was either cast in or placed on ceresin wax.

Complete electromagnetic shielding was necessary to obtain the desired steadiness.

Very short leads were found to be essential in the control-grid circuit of the pliotron, so that the capacity of that portion of the circuit was kept small. (Otherwise, the reaction of the circuit as a whole was very sluggish. This was particularly true when working at extreme sensitivity; i.e., when the resistance R was of the order of $10''$ ohms.)

The sensitivity of the circuit as a whole was varied by changing the magnitude of the resistance R in the control-grid circuit of the pliotron. A variable grid-bias was found to be necessary to keep the pliotron at the proper operating point on its plate current-grid potential characteristic when the filament of the photoelectric cell was operated at high temperatures.

The photoelectric current was measured by balancing out the effect produced by the thermionic current in the plate circuit of the pliotron, and recording the galvanometer deflection on illumination on the photosensitive surface as a measure of the photoelectric current.

The circuit was calibrated by applying a known potential to the control-grid of the pliotron and observing the corresponding galvanometer deflection. Then by knowing the magnitude of R, the ratio of this applied potential to R is numerically equal to the increase in the control-grid current. From these data, the current sensitivity of the system follows directly; *viz.*,

$$\text{Current Sensitivity} = \frac{\text{Change in the control grid current}}{\text{Galvanometer deflection (mm)}}.$$

The constancy of the resistance R is evidenced by the small drift observed; i.e., 1 or 2 mm/hr.

With this apparatus, photoelectric currents as small as 10^{-14} ampere could be measured to

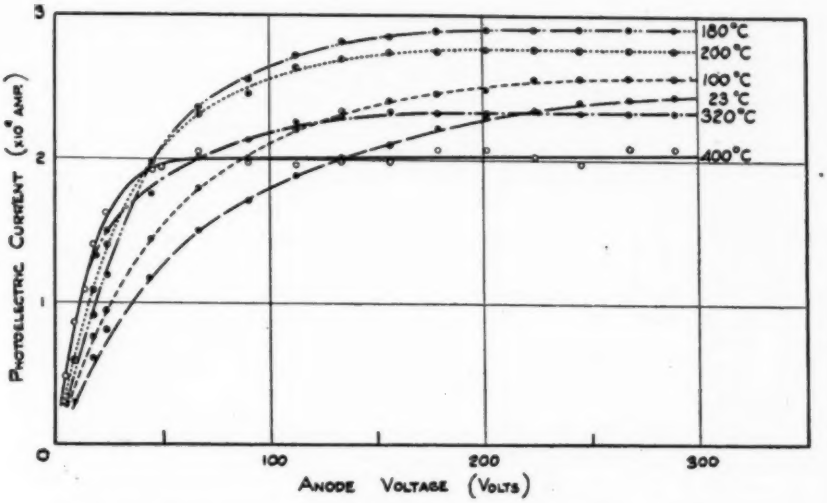

FIG. 3. Variation of photoelectric current with anode voltage.

within 1 percent; and the results could be repeated. Even greater sensitivity than this is possible with this set-up, but repetition of results is out of the question at present because of the number of factors that must be controlled simultaneously.

RESULTS

(1)

Various filters were interposed in the path of the incident light in order to determine which particular portion of the mercury arc spectrum was producing the observed photoelectric effect.

Measurements of the photoelectric current were made corresponding to a plate potential of 270 volts at two different filament temperatures (23°C and 180°C).

No photoelectric current could be observed under either of the above conditions when the incident light was filtered through either hard rubber or ordinary window glass.

Corning Filter No. 774, which transmits light throughout the range of approximately 2800A to above 30,000A, completely eliminated the photoelectric current under both of the above described conditions. While Corning Filter No. 980, which transmits light throughout the range of about 2300A to above 3000A allowed light to pass through which produced a photoelectric current corresponding to a deflection of 7.65 cm when the photoactive surface was at 23°C, and 14.1 cm when the photoactive surface was at 180°C. In both of these cases, the potential difference between the anode and the photoactive surface was 270 volts.

The results for these experiments indicate that the photoelectric effect observed was produced by light rays in the region between 2300A and 3100A; and for this reason was a true light effect rather than a temperature effect.

In order to determine this range more accurately, a quartz monochromatic-illuminator was substituted for the filters under the above conditions. When the filament was at 23°C, no photoelectric effect could be observed; but when the filament temperature was 180°C a deflection of 9 mm was produced by the 2536A line, while lines in the vicinity of 3000A produced a deflection of 2 mm. None of the other lines produced any effect that could be measured. These results agree qualitatively with those obtained with the filters. They were measured with a system current-sensitivity of the order of 10^{-16} amp./mm.

(2) The variation in the photoelectric current with the plate-potential for various temperatures

Fig. 3 shows the variation of the photoelectric current with the plate potential for a thorium-oxide-coated platinum filament. It will be noticed that both the saturation photoelectric current and the voltage required to produce saturation depend on the temperature. At

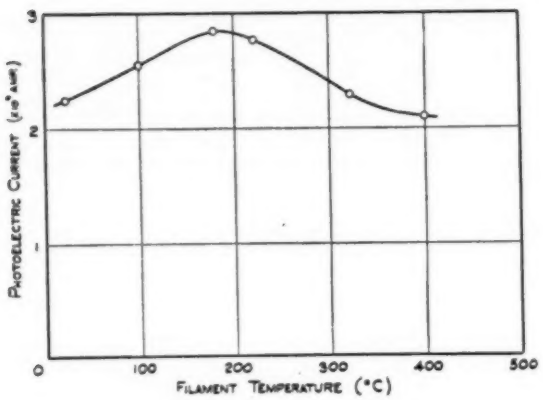

FIG. 4. Temperature variation of saturation photoelectric current.

higher temperatures, saturation is reached at a lower voltage.*

(3) The variation in the saturation-photoelectric-current with the temperature

The relation between the saturation photoelectric current and the temperature is given in Fig. 4. It evidently reaches a maximum in the neighborhood of 200°C, and then decreases up to about 400°C. Subsequent observations extending up to about 800°C seem to indicate that this curve goes through a minimum in the neighborhood of about 600°C and then increases. However, since considerable modification of apparatus would be necessary to extend the range of accurate measurement beyond the temperatures actually represented on the curve, no attempt has yet been made to confirm this.

At a temperature of 1066°C, the thermionic current from this cell was found to be saturated at 135 volts. The saturation thermionic current at this temperature was $2.6/10^{-7}$ ampere.

At temperatures higher than 180°C, the saturation thermionic current was always more than ten times as large as the corresponding saturation photoelectric current. For example, their ratio was 12.3 at 180°C and 36.0 at 400°C.

(4) Fatigue

No photoelectric fatigue could be observed from this cell over periods of about one hour. There is apparently no decrease in the photosensitivity of a thorium-oxide-coated platinum filament with thermionic saturation as was reported by both Crew[8] and Bodemann[9] for the case of platinum filaments coated with barium-oxide.

I am very deeply indebted to Professor Merritt of the Department of Physics of Cornell University for his many suggestions and for his continued interest in this problem and in my work.

* These data as represented in Fig. 3 were reproducible quantitatively during a particular experimental run. That is, corresponding to a given filament temperature the values of the photoelectric current taken with increasing anode voltage agreed with those values of the photoelectric current taken with decreasing values of the anode voltage. However, these values could not be reproduced quantitatively over different experimental runs because of variation from day to day of the light source intensity.

In spite of this, there seems to be a general qualitative agreement of all such data taken, the results as presented in Fig. 3 being typical.

Grating Theory and Study of the Magnetostatic Oscillator Frequency

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(Received February 27, 1935)

A special type of echelette grating for wavelength measurements of ultra-short electromagnetic waves is described. The one grating may be used over the entire wavelength range from 1 to 10 cm without loss of effectiveness due to an automatically changing groove form which keeps the direction of the center of the diffraction pattern fixed. The formula for the distribution of the energy diffracted from the grating is given and found to agree with the experimental values. The magnetostatic oscillator produces a band of frequencies unless it is adjusted for the conditions at which the tube gives the maximum output, in which case no conclusions as to the width of the band could be drawn as the resolution of the spectrometer was too low.

INTRODUCTION

A SPECIAL type of echelette grating has been used for wavelength measurements of short electromagnetic waves in the region of 1 to 4 cm.[1, 2] The formula for the diffraction pattern of a grating with a triangular groove has been treated by J. A. Anderson and C. M. Sparrow[3] and Jenny E. Rosenthal.[4] Certain changes in the known formula for the energy distribution are made necessary in order to take account of the unusual construction of the grating used. It is the purpose of this paper to point out these changes and the reason for making them, and to offer experimental data which are in agreement with the theory. Also some conclusions will be drawn as to the width of the band of frequencies produced by the magnetostatic oscillator under various adjustments.

CONSTRUCTION AND USE OF GRATING

The diffraction grating consisted of 18 elements of sheet aluminum each about 7.4 cm wide and 70 cm long, which were mounted in a frame and hinged so that they could be rotated with respect to the frame of the grating, each element rotating about one of its edges as an axis. A bar hinged to the other edge of the strips at the upper end made them rotate in unison. One of the elements was clamped so that the plane of the individual elements did not change as the grating was rotated, the effect being an echelette grating with a changing groove form. The spectrometer was completed by two large parabolic brass mirrors, the source being at the focus of one and the receiver at the focus of the other. The mirrors were stationary so that the angle between the incident and diffracted beams remained constant. The elements of the grating were adjusted so that they always remained perpendicular to the bisector of the angle between the incident and diffracted beam.

THEORY OF THE GRATING

The wavelength is given by the ordinary formula: $n\lambda = 2a \cos i \sin \omega$ where n is the order of the spectrum, λ the wavelength in cm, a the distance between elements (7.49 cm for the grating used), $2i$ the angle between the incident and the diffracted beam, and ω the angle through which the grating turns in passing from the position at which the direct image is received and the position at which the nth order is received.

The formula for the distribution of the energy diffracted from an ordinary echelette grating must be changed to take account of the following differences in construction and use:

(1) One side of the groove may be neglected. The width of one side of the groove is merely the thickness of the aluminum strip and the arrangement of the spectrometer is such that the radiation strikes this side very obliquely, therefore its contribution is negligible compared to the other side.

(2) The groove form depends upon the position of the grating. As the grating is rotated, the angle between the plane of the element and the plane of the grating changes. Also the effective

[1] Cleeton and Williams, Phys. Rev. **44**, 421 (1933).
[2] Cleeton and Williams, Phys. Rev. **45**, 234 (1934).
[3] Anderson and Sparrow, Astrophys. J. **33**, 338 (1911).
[4] Rosenthal, J. Opt. Soc. Am. **20**, 87 (1930).

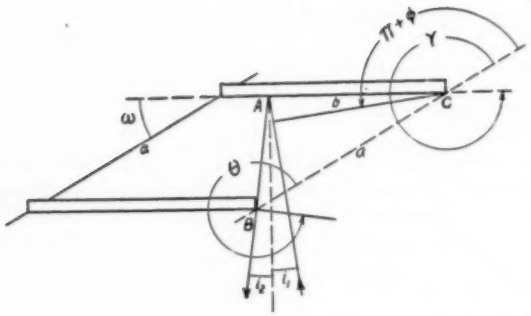

FIG. 1. Diagram of two elements of the grating. $\pi+\phi$, direction of the incident radiation; θ, direction of the diffracted radiation; γ, angle of the face of the element with the plane of the grating; a, the grating constant; b, the effective width of an element.

width of the element varies because of the shielding of the element by one of its neighbors.

(3) The angle between the incident and diffracted beams remains constant. This in connection with the changing groove form permits a simplification of the formula.

By following the notation used by Anderson and Sparrow[3] (see Fig. 1) the amplitude corresponding to one face, the effective width of which is b, is:

$$P=P_0(\sin u)/u$$

$$\text{where}\begin{cases} P_0=Ab\cos(\phi-\gamma), \\ u=(\pi b/\lambda)\sin(\phi-\gamma)+\sin(\theta-\gamma). \end{cases}$$

A is a constant depending upon the amplitude of the incident radiation. This expression may be simplified by a change in notation. Let i_1 be the angle between the normal to the element and the incident beam and i_2 the angle between the normal and the diffracted beam, then

$$\cos(\phi-\gamma)=\cos i_1,$$

$$b=a(\cos\omega-\sin\omega\tan i_2).$$

Since the grating was adjusted so that $i_1=i_2=i$, $P_0=A\ a\cos(\omega+i)$, and the expression for u becomes $u=(\pi a/\lambda)(\cos\omega-\sin\omega\tan i_2)(\sin i_1-\sin i_2)$.

But $$\lim_{i_1\to i_2}\sin u/u=1.$$

Therefore the amplitude corresponding to a single element is $P=A\ a\cos(\omega+i)$. The intensity of the radiation due to one element for any position of the grating denoted by the angle ω may be written, $I=C\cos^2(\omega+i)$, where C is a constant. The intensity due to all N elements, T, is evaluated in the ordinary way.

$$T=C\left[\cos(\omega+i)\frac{\sin((2\pi Na/\lambda)\sin\omega\cos i)}{\sin((2\pi a/\lambda)\sin\omega\cos i)}\right]^2. \quad (1)$$

The maximum value of the intensity is given by N^2P^2. If the grating is wider than the incident beam of radiation, the number of elements which are effective in producing the diffraction pattern will vary as the grating is rotated. It may be seen from Fig. 1 that the value of N will be inversely proportional to $\cos(\omega-i)$. Therefore the maximum intensity will be given by

$$T_m=C'(\cos^2(\omega+i)/\cos^2(\omega-i)), \quad (2)$$

where C' is a constant.

EXPERIMENTAL

An attempt was made to compare the observed diffraction pattern of the grating with the theoretical curve. The receiver was a galvanometer connected to a fixed iron pyrite crystal detector. The intensities were expressed in millivolts read from a curve which gave the d.c. voltage necessary to produce a difference between the currents in the two directions equal to the current in the receiving galvanometer. This assumes that the crystal acts the same at high frequencies as at low.

Eq. (2) gives the rate at which the intensity of the higher orders should fall off. The value of i was seven degrees for the spectrometer used. Table I gives some typical data showing how the intensities decrease with the order for several different wavelengths and also the intensity as calculated from the formula. The constant C'

TABLE I. *Typical data showing comparison between observed and calculated intensities in different orders.*

Order	λ=1.65 cm Obs.	Cal.	λ=1.76 cm Obs.	Cal.	λ=2.93 cm Obs.	Cal.	λ=4.11 cm Obs.	Cal.
1	82	82	56	58	37	36	144	148
2	78	78	54	54	32	32	122	122
3	73	73	52	51	28	28	81	77
4	67	68	49	48				
5	60	62						

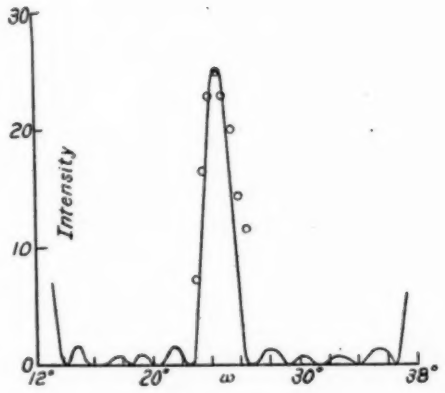

FIG. 2. The theoretical curve for the intensity of a six element grating, and some observed points taken in the second order.

was so chosen as to give the best agreement. The slight disagreements may easily be due to line voltage changes which change the intensity of the oscillator.

In order to compare the observed diffraction pattern with Eq. (1), one must know how many elements are effective. In order to make this comparison all elements except six were removed and the lower half of the mirror was shielded in order to eliminate the effect of the shadow of the electromagnet of the oscillator. The theoretical curve for the second order of a six-element grating is shown in Fig. 2, together with several observed points. The four secondary maxima were observed between the primary maxima but because of their overlapping and the uncertainty in the calibration of the receiver at such small intensities no comparison could be made.

It is important to know how wide a band of frequencies the magnetostatic oscillator produces. This range of frequencies may be due to poor adjustment of the oscillator or to fluctuations in the line voltages. First consider only the case where the oscillator is adjusted for maximum output. Such is the case for the data in Fig. 2. If the spectrometer, other than the grating, had perfect resolution, and the oscillator produced waves of only one frequency, the observed points should fall on the theoretical curve. If either of the above conditions is not satisfied, the observed peak should be wider. The observed peak is somewhat wider. However, considering the fact that the spectrometer slits have considerable width (exit slit was 2 cm wide) the divergence may easily be explained without assuming an appreciable range of frequencies produced by the oscillator.

When the oscillator is not adjusted for maximum output, the frequency becomes wider as is shown in Table II where the width of the intensity curve in waves per cm in the 5th order was measured for five values of the magnetic field. The line width was smallest when the adjustment was for maximum intensity. The third column gives the intensity at the maximum translated to a millivolt scale.

TABLE II. *Width of the intensity curve in the fifth order as measured for five values of the magnetic field.*

H (gauss)	λ (cm)	Int. (mv)	ν (cm^{-1})
7900	1.48	25	0.033
8300	1.39	34	0.032
8750	1.33	48	0.021
9300	1.27	28	0.033
9700	1.21	30	0.040

Since the oscillator tube was so small, no entrance slit was used. The effective width of the source would be difficult to estimate but would certainly have a maximum equal to the distance between the pole pieces of the electromagnet which produced the field for the oscillator tube. The way in which the width of the exit slit affects the width and maximum intensity of the intensity curve is shown in Table III. A plane reflector was substituted for the grating and a wavelength of 1.32 cm was used. The area under the intensity curve was found to be approximately proportional to the width of the slit. The variations may easily be due to changes in the output of the oscillator since it requires several minutes to obtain one curve.

TABLE III. *Effect of the width of the exit slit on the width and maximum intensity of the intensity curve.*

Exit slit width (cm)	Angular width of curve	Maximum intensity	Area under curve
4	2° 35′	43	702
3	2° 15′	38	547
2	2° 0′	35	444
1	1° 30′	21	197

I wish to express my appreciation to Professors N. H. Williams, E. F. Barker and C. F. Meyer of the University of Michigan who made many valuable suggestions.

Notice to Contributors

Manuscripts

Send manuscripts to John T. Tate, University of Minnesota, Minneapolis, Minnesota.

Papers must be in English, typewritten double spaced with wide margins.

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An abstract must accompany each article. It should be adequate as an *index* and as a *summary*. As an index it should give all subjects, major and minor, concerning which new information is presented. As a summary it should give the conclusions of the article and all numerical results of general interest.

References

References should appear as footnotes, numbered consecutively, and arranged thus:

A. B. Smith, Phys. Rev. **41**, 852 (1932).

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